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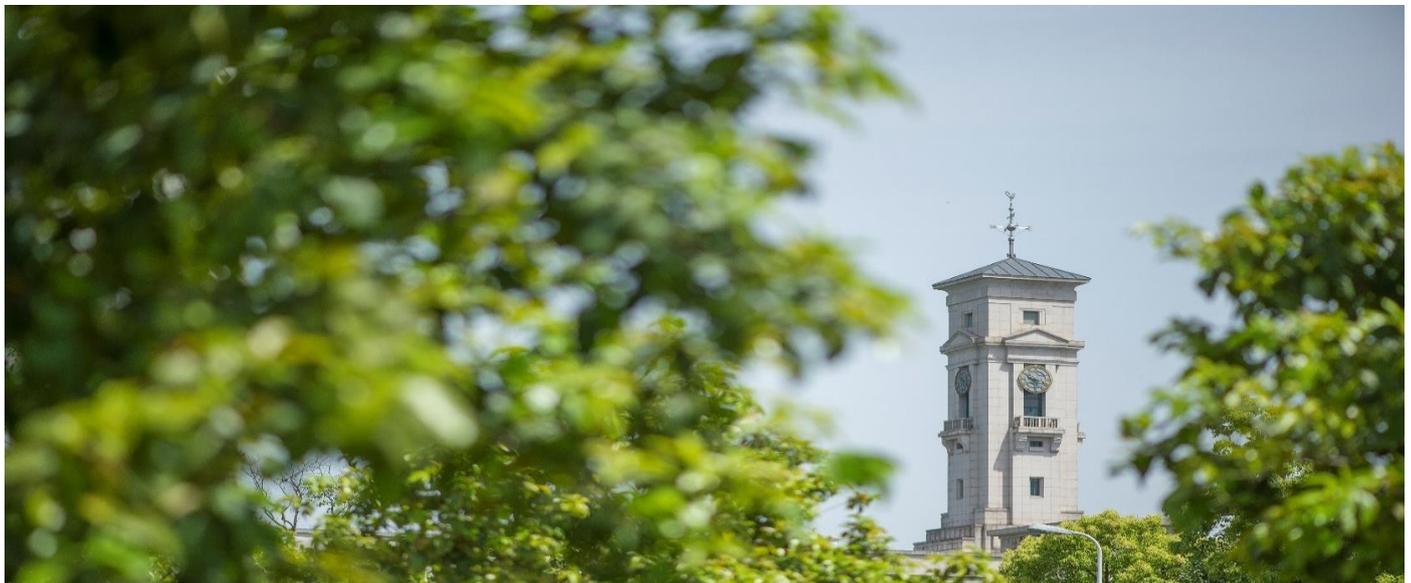
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Formulation and solution of a two-stage capacitated facility location problem with multilevel capacities

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Abstract In this paper, the multi-product facility location problem in a two-stage supply chain is investigated. In this problem, the locations of depots (distribution centres) need to be determined along with their corresponding capacities. Moreover, the product flows from the plants to depots and onto customers must also be optimised. Here, plants have a production limit whereas potential depots have several possible capacity levels to choose from, which are defined as multilevel capacities. Plants must serve customer demands via depots. Two integer linear programming (ILP) models are introduced to solve the problem in order to minimise the fixed costs of opening depots and transportation costs. In the first model, the depot capacity is based on the maximum number of each product that can be stored whereas in the second one, the capacity is determined by the size (volume) of the depot. For large problems, the models are very difficult to solve using an exact method. Therefore, a matheuristic approach based on an aggregation approach and an exact method (ILP) is proposed in order to solve such problems. The methods are assessed using randomly generated data sets and existing data sets taken from the literature. The solutions obtained from the computational study confirm the effectiveness of the proposed matheuristic approach which outperforms the exact method. In addition, a case study arising from the wind energy sector in the UK is presented.

Key words: Facility location, matheuristic, ILP.

1. Introduction

The two-stage facility location problem (TSFLP) with two types of facilities can be classified as a type of hierarchical facility location problem. In the first stage, products produced/supplied by plants are transferred to capacitated depots. The location and the number

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of plants and depots can be treated as fixed or as decision variables. Their capacity may be finite (capacitated) or unlimited (uncapacitated). In the second stage, the products are delivered to customers. The problem to be addressed includes finding an optimal distribution structure in order to minimise both the fixed (opening) cost of the plants and depots and the transportation costs associated with both stages.

The two-stage location problem has been investigated in the literature. A dual-based optimization procedure for the two-echelon uncapacitated facility location problem was proposed by Gao and Robinson (1992). The two-stage facility location with a single sourcing constraint on depot-plant assignment and customer-depot assignment was investigated by Tragantalerngsak et al. (1997) where six Lagrangean relaxation heuristics are introduced. Marín & Pelegrín (1999) applied Lagrangian relaxation to the resolution of two-stage location problems. Klose (1999 and 2000) studied the two-stage facility location with a single product, depot location, plant-depot multiple source flow and single source customer-depot assignment. An effective linear programming approach and a Lagrangean relax-and-cut algorithm are proposed to achieve lower and upper bounds for the problem. Tragantalerngsak et al. (2000) also proposed a Lagrangean-based branch-and-bound method to solve the problem. Hinojosa et al. (2000) studied a heuristic algorithm based on Lagrangean relaxation to solve a multi-period two-echelon multicommodity capacitated plant location problem.

Keskin & Üster (2007a and 2007b) proposed a scatter search for a multi-type transhipment point location problem with multi-commodity flow and studied meta-heuristic approaches with memory and evolution for a multi-product production/distribution system design problem respectively. Li et al. (2011) proposed a Lagrangean-based heuristic for a two-stage facility location problem with handling costs with multiple products and three layers of nodes: plants with limited production capacities, capacitated depots to be located and customers with known demands per product. The aim of their model is to minimize a total cost comprising depot opening, transportation and handling costs. Li et al. (2014) investigated a multi-product facility location problem in a two-stage supply chain in which plants have a production limit, potential depots have limited storage capacity and customer demands must be satisfied by plants via depots. A hybrid method is developed where the initial lower and upper bounds are obtained by a Lagrangean based heuristic and a weighted Dantzig–Wolfe decomposition and path-relinking combined method are applied to improve obtained bounds. Several variants of the two-stage location problem were also studied by Li *et al.* (2012), Rodríguez *et al.* (2014), Camacho-Vallejo *et al.* (2015), and Mišković and Stanimirović (2016).

The papers cited above deal with the two-stage facility location problem with fixed capacity for each potential depot. However, in practical situations, the capacity of the depot is also considered as a decision variable which needs to be determined (Correia and Captivo, 2003). This means that the problem is not only to find the optimal location of the depot but also its capacity. In this study, we not only deal with one product but also with multiple products meaning that a depot may have a different capacity for each product. To the best of our knowledge, this type of problem has not yet been addressed in the literature. Therefore, this paper proposes new mathematical models and a solution method to deal with the two-stage capacitated facility location problem in the presence of multilevel capacities.

The main contributions of this paper are as follows:

- Propose for the first time mathematical models for the two-stage capacitated facility location problem in the presence of multi-product and multilevel capacities,
- Propose an effective matheuristic approach based on an aggregation method to solve the problem,
- Provide a new dataset for the new problem and produce good quality solutions for benchmarking purposes.

The remainder of this paper is organised as follows. Mathematical models for the two-stage capacitated facility location problem considering the presence of multilevel capacities are presented in Section 2. Section 3 discusses the proposed matheuristic approach to solve the problem. The computational results are presented in Section 4 followed by conclusions in the final section.

2. Problem Formulation

In this section, two mathematical models of the two-stage capacitated facility location problem considering the presence of multilevel capacities are presented. Here, a set of potential depots to choose from is given where the depots to be opened (opened depots) can be determined by solving the models. In the first model, which we refer to as Model A, each potential depot has an associated set of possible capacities for storing each product with different fixed costs. The capacity is related to the maximum amount of units that can be stored for each product in the depot. In this paper, we refer to this capacity as the '*product capacity*'. For example, suppose that there are 2 products (Product P1 and P2) where the possible

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capacities of a depot for these products are 10, 15 and 30 for Product P1 and 80, 120 and 160 for product P2. Here, the first decision is to determine whether we will open this depot or not. The second is to decide whether both products will be stored in this depot. The depot may store both products P1 and P2 or just one of them. Finally, the optimal capacity for each product for this depot needs to be found.

In the second proposed model, termed Model B, the capacity of a depot is based on the size (volume) of the depot that is required to be built. In this model, a set of possible capacities (volume) for each potential depot is given. In this study, we refer to this capacity as the '*volume capacity*'. The first decision generated by this model is to determine whether the depot should be opened or not whilst the second is to decide how big a depot needs to be built. Here, we assume that the volume needed to store one unit product is known. The total volume needed to store all products must not exceed the size of the opened depot.

2.1 Model A

In model A, there are two types of fixed (opening) cost where the first is the setup (fixed) cost for opening a depot. The second fixed cost is related to the capacity of each product used in the depot. The fixed cost is dependent on the product capacity and the location of the depot. Therefore, the fixed cost of a potential depot may be different from that of others. An opened depot is also not necessarily built to store all products. In other words, the opened depot may keep only selected products. In this model the first total fixed cost can be determined using decision variable Q_j and the second one by $\hat{Y}_{jpd}$. The following notations are used to describe the sets, parameters, and decision variables of Model A.

Sets

I : set of plants with i as its index and $l = |I|$

J : set of potential depots with j as its index and $m = |J|$

K : set of customers with k as its index and $n = |K|$

P : set of products with p as its index and $o = |P|$

D_{jp} : set of product capacities at potential depot j for storing product p with d as its index

   and $\delta_{jp} = |D_{jp}|$.

Parameters

s_{ip} : the capacity of plant i ($i \in I$) to produce product p ($p \in P$)

w_{kp} : the demand of customer k ($k \in K$) for product p ($p \in P$)

$\tilde{f}_j$: the fixed cost for opening depot j ($j \in J$)

$\hat{f}_{jpd}$: the fixed cost to store product p ($p \in P$) using product capacity d ($d \in D_{jp}$) in depot

       j ($j \in J$)

$\hat{b}_{jpd}$: the number of product p ($p \in P$) that can be stored in depot j ($j \in J$) when using

       product capacity d ($d \in D_{jp}$)

c_{ijp} : unit transportation cost of product p ($p \in P$) from plant i ($i \in I$) to depot j ($j \in J$)

$\hat{c}_{jkp}$: unit transportation cost of product p ($p \in P$) from depot j ($j \in J$) to customer k

       ($k \in K$)

Decision Variables

X_{ijp} : the amount of product p ($p \in P$) transported from plant i ($i \in I$) to depot j ($j \in J$)

$\hat{X}_{jkp}$: the amount of product p ($p \in P$) transferred from depot j ($j \in J$) to customer k

       ($k \in K$)

$\hat{Y}_{jpd}$ $= 1$, if depot j ($j \in J$) uses product capacity d ($d \in D_{jp}$) to store product p ($p \in P$) or

       $= 0$ otherwise

Q_j $= 1$, if depot j ($j \in J$) is open (selected) or

       $= 0$ otherwise

The problem can be modelled as an integer linear problem (ILP) as follows.

$$Min \sum_{j \in J}\left(\tilde{f}_j \cdot Q_j\right) + \sum_{j \in J}\sum_{p \in P}\sum_{d \in D_{jp}}\left(\hat{f}_{jpd} \cdot \hat{Y}_{jpd}\right) + \sum_{i \in I}\sum_{j \in J}\sum_{p \in P}\left(X_{ijp} \cdot c_{ijp}\right) + \sum_{j \in J}\sum_{k \in K}\sum_{p \in P}\left(\hat{X}_{jkp} \cdot \hat{c}_{jkp}\right) \quad (1)$$

Subject to

$$\sum_{j \in J}X_{ijp} \leq s_{ip}, \quad \forall i \in I, p \in P \quad\quad\quad (2)$$

$$\sum_{i \in I}X_{ijp} \leq \sum_{d \in D_{jp}}\hat{b}_{jpd} \cdot Y_{jpd}, \quad \forall j \in J, p \in P \quad\quad\quad (3)$$

$$\sum_{d=1}^{\delta_{jp}} Y_{jpd} \leq Q_j, \quad \forall j \in J, p \in P \quad\quad\quad (4)$$

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$$\sum_{j \in J} \hat{X}_{jkp} = w_{kp}, \qquad \forall k \in K, p \in P \tag{5}$$

$$\sum_{i \in I} X_{ijp} = \sum_{k \in K} \hat{X}_{jkp}, \qquad \forall j \in J, p \in P \tag{6}$$

$$X_{ijp} \geq 0, \text{ integer} \quad \forall i \in I, j \in J, p \in P \tag{7}$$

$$\hat{X}_{jkp} \geq 0, \text{ integer} \quad \forall j \in J, k \in K, p \in P \tag{8}$$

$$Y_{jpd} \in \{0,1\}, \qquad \forall j \in J, p \in P, d \in D_{jp} \tag{9}$$

$$Q_j \in \{0,1\}, \qquad \forall j \in J \tag{10}$$

In the objective function (1), the first term represents the fixed cost of opening the depots, the second term is the fixed cost of the depots to store the products, the third term is the total transportation cost from the plants to the depots and the fourth term is the total transportation cost from the depots to the customers. Constraints (2) ensure that the total number of products transferred from a supplier does not exceed its capacity. Constraints (3) guarantee that the capacity constraints at the depots are satisfied. Constraints (4) indicate that each opened depot only uses at most one capacity level for each product. Constraints (5) ensure that the demand of each customer for each product is met. Constraints (6) state flow conservation constraints for the depots. Constraints (7) and (8) impose non-negativity and integer conditions on the number of products delivered. Constraints (9) and (10) refer to the binary nature of the variables Y and Q (the decisions whether a depot is opened or not and which capacity is used by the opened depot).

2.2 Model B

In the second model, model B, the capacity considered is based on the required size (volume) of the depot. In contrast to Model A, the fixed cost in Model B only consists of one term as the fixed cost includes those of both its opening and the storing of products, based on its capacity. In this model the total fixed cost can hence be calculated based on variable decision Y_{jd}. Several possible volume capacities for each depot are considered in this model where the volume capacity of a potential depot has a fixed cost that is also dependent on its size and location. The dimension/volume of each product is required in this model in order to determine the capacity constraints of the depots. The notations used for sets and parameters in this model are similar to the ones provided in the previous model (model A) with some revisions described

as follows. Set D_{jp} is replaced by $\hat{D}_j$ whereas parameters $\hat{f}_{jpd}$ and $\hat{b}_{jpd}$ are substituted by f_{jd} and b_{jd} respectively. Parameter $\tilde{f}_j$ is not required in this model but parameter α_p is added.

Sets

$\hat{D}_j$: set of feasible volume capacities at potential depot j with d as its index and

$$\hat{\delta}_j = \left|\hat{D}_j\right|.$$

Parameters

f_{jd} : the fixed cost for opening depot j ($j \in J$) using volume capacity d ($d \in \hat{D}_j$)

b_{jd} : the volume (size) of depot j ($j \in J$) using volume capacity d ($d \in \hat{D}_j$)

α_p : the volume required to store a unit of product p ($p \in P$)

Decision Variables

X_{ijp} and $\hat{X}_{jkp}$ as defined in the previous model.

Y_{jd} $= 1$, if depot j ($j \in J$) uses volume capacity d ($d \in \hat{D}_j$) or $= 0$ otherwise

The problem can be modelled as an integer linear problem as follows.

$$Min \sum_{j \in J} \sum_{d \in \hat{D}_j} \left(f_{jd} \cdot Y_{jd}\right) + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \left(X_{ijp} \cdot c_{ijp}\right) + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \left(\hat{X}_{jkp} \cdot \hat{c}_{jkp}\right) \tag{11}$$

Subject to

$$\sum_{j \in J} X_{ijp} \le s_{ip}, \quad \forall i \in I, p \in P \tag{12}$$

$$\sum_{i \in I} \sum_{p \in P} X_{ijp} \cdot \alpha_p \le \sum_{d \in \hat{D}_j} Y_{jd} \cdot b_{jd}, \quad \forall j \in J \tag{13}$$

$$\sum_{d \in \hat{D}_j} Y_{jd} \le 1, \quad \forall j \in J \tag{14}$$

$$\sum_{j \in J} \hat{X}_{jkp} = w_{kp}, \quad \forall k \in K, p \in P \tag{15}$$

$$\sum_{i \in I} X_{ijp} = \sum_{k \in K} \hat{X}_{jkp}, \quad \forall j \in J, p \in P \tag{16}$$

$$X_{ijp} \ge 0, \text{ integer} \quad \forall i \in I, j \in J, p \in P \tag{17}$$

$$\hat{X}_{jkp} \ge 0, \text{integer} \quad \forall j \in J, k \in K, p \in P \tag{18}$$

$$Y_{jd} \in \{0,1\}, \qquad \forall j \in J, d \in \hat{D}_j \tag{19}$$

The objective function (11) aims to minimise the sum of the fixed costs of opening depots and transportation costs. The first term of this objective function represents the fixed cost of opening the depots based on the capacity of the depots, the second term is the total transportation cost from the plants to the depots, and the third term is the total transportation cost from the depots to the customers. Constraints (12) enforce the capacity constraints of the plants. Constraints (13) ensure that the size (volume) of depots is enough to store the products. Constraints (14) make sure that each opened depot only uses one volume capacity. Constraints (15) guarantee that the demand of each customer is satisfied. Constraints (16) state flow conservation constraints for the depots.

3. The solution method

The classical two-stage capacitated facility location problem (TSCFLP) is an NP-hard optimization problem as it represents a generalization of the simple plant location problem, which is proved to be NP-hard by Krarup and Pruzan (1983). The proposed model with multilevel capacities is even harder to solve than the classical TSCFLP using an exact method (via an optimizer software such as CPLEX, Lindo, and Xpress) especially when the size of the problem is relatively large. To overcome this weakness a matheuristic approach is developed by integrating an aggregation technique and an exact method. We refer to this method as a MAAT (Matheuristic Approach incorporating an Aggregation Technique).

When the location problems involve a large number of demand points, it may be sometimes impossible and time consuming to solve to optimality (Francis *et al*., 2009). It is quite common to aggregate demand points/depots when solving large scale location problems. The main idea behind the aggregation is to simplify the problem by reducing the number of demand points/depots to be small enough that an optimiser can be used to solve the reduced problem within a reasonable amount of computing time. However, the approximation involved may lead to a level of sub-optimality when the aggregated solution is put into practice in the actual real-world situation. The aggregation technique has successfully addressed large facility location problems such as for large *p*-median (Irawan *et al*., 2014; Irawan and Salhi, 2015a) and *p*-centre problems (Irawan *et al*., 2016). A review on aggregation techniques for large facility location problems is provided by Irawan and Salhi (2015b).

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The proposed matheuristic approach (MAAT) is developed to solve both Models A and B presented in Section 2. Matheuristics have been successfully used to solve tackle facility location problems (Stefanello *et al.*, 2015; and Irawan *et al.*, 2017). The proposed method consists of three stages where the main steps of this approach are depicted in Figure 1. The first stage is an iterative process that incorporates the aggregation of potential depot sites and the implementation of the proposed local search. Firstly, m potential depot sites are aggregated into μ potential sites, with $\mu << m$. The value of m is determined based on the maximum number (upper bound) of the facilities that need to be opened (ρ). The value of ρ can be approximated by following expressions:

$$\rho = \underset{p \in P}{\text{Max}} \left(\frac{\sum\limits_{k \in K} w_{kp}}{\underset{j \in J}{\text{Min}} \left(\underset{d \in D_{jp}}{\text{Min}} \left(\hat{b}_{jpd} \right) \right)} \right) \text{ for Model A} \tag{20}$$

$$\rho = \underset{p \in P}{\text{Max}} \left(\frac{\sum\limits_{k \in K} \left(w_{kp} \cdot \alpha_p \right)}{\underset{j \in J}{\text{Min}} \left(\underset{d \in \hat{D}_j}{\text{Min}} \left(\hat{b}_{jd} \right) \right)} \right) \text{ for Model B} \tag{21}$$

Here, $\mu = \beta \cdot \rho$ where β is a parameter. When choosing the aggregated potential depot sites, the aggregation includes the depot sites obtained from the previous iteration (the best solution, S^*). The remaining $(\mu - |S^*|)$ potential depots are randomly chosen from the m potential depot sites. The main idea behind this is to make sure that the reduced problem has a feasible solution. The resulting aggregated problem with l plants, μ depots and n customers is then solved by CPLEX within τ seconds. A duality gap (%Gap) is also set as a termination criterion where CPLEX will stop when the %Gap reaches ε%. Let Z be the terminating objective function value and S be the corresponding vector of the obtained facility configuration. The description of the proposed local search is presented in the following subsection. The obtained depots location configuration, if it is better than the previous one, is then fed to the next iteration as part of the set of the aggregated potential depot sites. The process is repeated T times and the best solution (S^*) from this step will be fed to the next step. The values of β and T influence the quality of the solution obtained. The chance of getting a better solution is higher when the values of β and T are set higher as this will increase diversification. However, the computational time also increases for higher values of β and T.

Initialisation

Define T, β, τ, ε and τ'. Set $Z^* = \infty$ and $S* = \emptyset$.

Stage 1

1. Find the maximum number (upper bound) of the facilities that need to be opened (ρ).

2. Set $\mu = \beta \cdot \rho$

3. Execute the following step T times:
   a. Aggregate m to μ potential facility sites using a random approach and by including the facility locations in the incumbent solution ($S*$).
   b. Solve the aggregated problem using the exact method (CPLEX) within τ seconds. A duality gap (%Gap) is also set as a termination criterion where CPLEX will stop when the %Gap reached ε%. Let Z be its objective function value with S as vector of the obtained facility configuration.
   c. If $Z < Z^*$ then set $Z^* = Z$ and $S^* \leftarrow S$.

Stage 2

Apply the proposed local search on the original problem using Z^* and S^* obtained from Stage 1 as the initial solution. In other words, we call LocalSearch (Z^* and S^*).

Stage 3

Implement the exact method (CPLEX) to solve model 1-11 (for solving Model A) and 12-20 (for solving Model B) within τ' seconds using the obtained $|S*|$ depot locations from the previous stage. In other words, in the model the set of potential depot sites (J) is replaced by $S*$. The model will find the optimal capacity for each depot and the objective function value Z^*.

Figure 1. The proposed matheuristic approach (MAAT)

In the second stage (Stage 2), the proposed local search is applied to solve the original problem (without aggregation) starting from the best depot configuration obtained from the previous stage. The obtained solution ($S*$) from the local search on the original problem is then fed into the final stage where the mathematical formulation of Model A or Model B is solved by an exact method (CPLEX). In the final stage, when solving Model A and Model B, the number of potential depots is reduced from m to $|S*|$. In other words, the set of potential depot sites (J) is replaced by $S*$ (the incumbent solution). CPLEX will find the best location to open the depots (if necessary), determine the best capacity for each opened depot, the products flows

(integer) from plants to depots and from depots to customers, and calculate the objective function value (Z^*).

The proposed local search

The proposed local search is a hybridisation of the fast interchange heuristic proposed by Whitaker (1983) and an exact method. We enhance the heuristic by incorporating the exact method to solve the multi-product capacitated transhipment problem (MPTP). The exact method is integrated within the local search to optimally solve the transhipment problem whenever the locations of opened depots along with their capacity are known/fixed. Moreover, we also enhance this heuristic by replacing a depot in the current solution with the potential depot (not in current solution) that is not too far from the removed depot. By restricting the search, the local search runs relatively fast at the expense of a small loss in quality.

For Model A, in the case where the location of the opened depots along with their capacity for each product are known, the problem can be treated as the multi-product capacitated transhipment problem, which we refer to as the MPTP-A. The MPTP-A is also relatively easy to solve when we relax the amount of products transported from one node to others to a real value instead of an integer one. The MPTP will hence be a linear programming formulation. Let $S \subset J$ be the set of opened depots and a_{jp} be the product capacity used by depot j to store product p. The mathematical model for the MPTP-A is as follows:

<u>Decision Variables</u>

X_{ijp} : the amount of product p ($p \in P$) transported from plant i ($i \in I$) to depot j ($j \in J$)

$\hat{X}_{jkp}$: the amount of product p ($p \in P$) transferred from depot j ($j \in J$) to customer k

   $(k \in K)$

The MPTP-A can be modelled as a linear problem as follows.

$$Min \ \sum_{j \in S} \widetilde{f}_j + \sum_{j \in S} \sum_{p \in P} \hat{f}_{jp} a_{jp} + \sum_{i \in I} \sum_{j \in S} \sum_{p \in P} \left(X_{ijp} \cdot c_{ijp} \right) + \sum_{j \in S} \sum_{k \in K} \sum_{p \in P} \left(\hat{X}_{jkp} \cdot \hat{c}_{jkp} \right) \tag{22}$$

Subject to

$$\sum_{j \in S} X_{ijp} \leq s_{ip}, \quad \forall i \in I, p \in P \tag{23}$$

$$\sum_{i \in I} X_{ijp} \leq \hat{b}_{jp} a_{jp}, \quad \forall j \in S, p \in P \tag{24}$$

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$$\sum_{j \in S} \hat{X}_{jkp} = w_{kp}, \qquad \forall k \in K, p \in P \tag{25}$$

$$\sum_{i \in I} X_{ijp} = \sum_{k \in K} \hat{X}_{jkp}, \qquad \forall j \in S, p \in P \tag{26}$$

$$X_{ijp} \geq 0, \qquad \forall i \in I, j \in S, p \in P \tag{27}$$

$$\hat{X}_{jkp} \geq 0, \qquad \forall j \in S, k \in K, p \in P \tag{28}$$

For Model B, similarly to Model A, when the location of the opened depots along with their capacities are fixed, the problem can also be treated as the multi-product capacitated transhipment problem which we refer to as the MPTP-B. S is also denoted as the set of opened depots with $\hat{a}_j$ be the volume capacity used by the depot j. The mathematical model for the MPTP-B is relatively similar to the one for the MPTP-A with minor revisions in objective function (22) and constraints (24). In the MPTP-B, the objective function (22) is replaced by objective function (29) as follows:

$$Min \quad \sum_{j \in S} f_{j\hat{a}_j} + \sum_{i \in I} \sum_{j \in S} \sum_{p \in P} \left(X_{ijp} \cdot c_{ijp} \right) + \sum_{j \in S} \sum_{k \in K} \sum_{p \in P} \left(\hat{X}_{jkp} \cdot \hat{c}_{jkp} \right) \tag{29}$$

and constraints (24) are replaced by constraints (30) as follows:

$$\sum_{i \in I} \sum_{p \in P} \left(X_{ijp} \cdot \alpha_p \right) \leq b_{j\hat{a}_j}, \qquad \forall j \in S \tag{30}$$

The main steps of the proposed local search are given in Figure 2 which is based on the fast interchange heuristic using a first improvement strategy (the exchange process is conducted once there is an improvement). The main objective of the algorithm is to seek a potential depot to be swapped with a one in the current solution where the swap process will be performed if there is an improvement. In this local search, when solving the transhipment problem, all potential depots (J) are imposed to use the largest capacity for storing each product (for Model A) or the volume (size) of the depot (for Model B).

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Procedure LocalSearch (Z and S)

1. Define $\hat{\tau}$ and γ.

2. Solve the MPTP-A (for Model A) or the MPTP-B (for Model B) optimally using CPLEX using S as set of opened facilities, each utilising its largest capacity. Let Z denote the corresponding objective function value.

3. While CPU time $< \hat{\tau}$ do the following steps:

    i.   Set $\theta = 0$ (θ is the saving occurred from swapping)

    ii.   For each potential depot j that is not in the solution ($j \in J, j \notin S$), find the nearest opened depot (in S). Let N_S be the set of potential depots where opened depot s ($s \in S$) is their nearest one.

    iii.   For each opened depot s ($s \in S$) determine $\hat{d}_s = \max_{j \in N_s}(\tilde{d}_j), \forall s \in S$ where $\tilde{d}_j$ is the distance between potential depot j and the nearest opened depot.

    iv.   For each potential depot j ($j \in J, j \notin S$), do the following:

        For each opened depot s ($s \in S$), do the following:

        If $d_{js} < \gamma \cdot \hat{d}_s$ then do the following procedure:

            a.   Set $S' \leftarrow S$ and remove facility s and insert facility j in set S' ($S'(s) \leftarrow j$)

            b.   Solve the MPTP-A (Model A) or MPTP-B (Model B) optimally using CPLEX with S' is the set of opened depots where each opened facility utilises the largest capacity. Let Z' denote its objective function value.

            c.   Calculate $\theta \leftarrow Z - Z'$

            d.   If $\theta > 0$ do the followings:

                - Update $Z \leftarrow Z'$ and $S \leftarrow S'$

                - Go to Step 3(i).

        End If

        End for s

        End for j

    v.   If $\theta \leq 0$ then go to Step 4

4. Return Z and S.

Figure 2. The main steps of the proposed local search

In this local search, we firstly set the maximum computational time ($\hat{\tau}$) to execute the local search. The transhipment problem (MPTP-A and MPTP-B) using the initial solution (S)

obtained from the previous step is then optimally solved to evaluate the quality of the solution (Z). In Steps 3(ii) – 3(iii) of Figure 2, the algorithm aims to allocate the potential depots (not in the current solution) to their nearest opened depot (incumbent solution). The longest/maximum distance $(\hat{d}_s)$ between the opened depots and their associated potential depots is then determined. The main idea behind this is to restrict the search (the swapping process) by imposing the condition that the substituted depot location must lie within a certain covering radius $(\gamma \cdot \hat{d}_s)$ from the opened depot that will be removed. This will make the local search runs more efficient (in terms of computing time) as the swap process will be skipped when the substituted depot is relatively far from the opened depot that will be removed. In Step 3(iv)a, potential depot j ($j \in J$, $j \notin S$) is inserted into the solution whereas opened depot s in the current solution is removed. Then, the transhipment problem (MPTP-A and MPTP-B) using S' is solved to optimality. In Step 3(iv)d, the swap will be conducted if there is an improvement. The local search will terminate if there is no improvement after all possible swaps based on the incumbent solution have been completed or if the computing time reaches $\hat{\tau}$ seconds.

4. Computational study

A set of computational experiments have been carried out to evaluate the performance of the proposed solution method. The proposed method was implemented/coded in C++ .Net 2012 where the IBM ILOG CPLEX version 12.6 Concert Library is used to solve the problems with exact method. The tests were run on a PC with an Intel Core i5 CPU @ 3.20GHz processor and 8.00 GB of RAM. In the computational experiments, two types of dataset are used. The first dataset (Dataset 1) is randomly generated to evaluate our solution method's ability to solve the two-stage capacitated facility location problem considering the presence of multilevel capacities for both models A and B. The second dataset is constructed based on the datasets from the literature and the wind energy industry in the UK. For the case study on the wind industry, the proposed model will be implemented to find the optimal locations for depots required for storing spare parts to support the operation and maintenance of offshore/onshore wind farms.

To evaluate the performance of the proposed matheuristic approach (MAAT), we compare the solutions obtained by the proposed method with those of the exact method (using IBM

ILOG CPLEX version 12.63). As the problem is very hard to solve to optimality, the computational time (CPU) for solving the problem using the exact method (CPLEX) is limited to 3 hours so the lower bound (LB) and upper bound (UB) can be attained. The performance of the proposed matheuristic method will be measured by %Gap between the Z value attained by the matheuristic approach and the lower bound (LB) obtained from the exact method. Moreover, the %Gap is also set as a termination criterion where CPLEX will stop when the %Gap reached 0.01%. %Gap is calculated as follows:

$$\%Gap = \frac{Z_m - LB}{Z_m} \times 100 \tag{31}$$

where Z_m refers to the feasible solution cost obtained by either the exact method (UB) or the proposed matheuristic approach. In the matheuristic approach, the parameters are set to the following values: $T = 10, \beta = 1.5, \tau = 150$ seconds, $\varepsilon = 0.5\%$, $\tau' = 108$ seconds, $\hat{\tau} = 0.25 \cdot n \cdot |P|$ seconds and $\gamma = 2.5$. For solving Model B, we set the value of β to 2 for $|P| = 10$ and to 3 for $|P| = 5$. Those parameters were selected based on a small preliminary study. This selection yields an acceptable performance with respect to the quality of the solution and the computational effort.

4.1. Experiments on the randomly generated data (Dataset 1)

In order to conduct extensive computational experiments, we generate a new dataset which we refer to as Dataset 1. This dataset consists of two instances, namely Instance 1A and Instance 1B. Instance 1A is used to evaluate the performance of our method when solving Model A whereas Instance 1B is for Model B. For Instance 1A, there are 20 problems to solve whereas Instance 1B consists of 15 problems. We set the number of products $|P|$ to 5 or 10. The number of plants (l) is varied between 5 to 25 with an increment of 5 whereas the value of m from 50 to 500 with an increment of 50. The number of customers (n) is set to $2m$ with the demand of each customer randomly generated between 1 and 5 for each product. The location of plants, warehouses and customers are generated randomly using a uniform distribution where $\Re^2 (n \times n)$ and the coordinates values are integer. The capacity of a plant for each product (s_{ip}) is generated based on the customer demand. It is assumed that there are three possible capacities for each product for each potential depot

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$(\left| D_{jp} \right| = \left| \hat{D}_j \right| = 3; j = 1,..., m; p \in P)$. We also generate the capacities (capacity/size) of each potential depot for each product ($\hat{b}_{pd}$ and b_{jd}) and its fixed (opening) cost along with the transportation cost per km per product. Here, we construct the dataset in such way that in a good solution, the total opening cost is close to the total transportation cost.

Computational Results on Instance 1A

The computational results on Instance 1A are presented in Tables 1 and 2, which show the computational results using the exact method (CPLEX) and the proposed matheuristic approach respectively. According to the tables, the complexity of the problem increases when the size of the problem increases as shown by the %Gap value. It is worth noting that when the number of products ($\left| P \right|$) is higher, the problem is more difficult to solve. According to the results shown in Table 1, the problems with $\left| P \right| = 5$ and $n \leq 200$ were relatively easily solved by the exact method. In these problems, the %Gap between UB and LB obtained is the requested %Gap termination criterion for CPLEX to solve the problem (i.e. a %Gap of 0.01%). In other words, CPLEX terminated before time based termination criterion of 3 hours.

Using the exact method, the %Gap value produced is relatively very high when $n \geq 700$. On average, the exact method yielded a %Gap of 20.64% which is considered as a large value. The exact method also produced the average portion of fixed (opening) costs of 61.98%. The proposed matheuristic method (MAAT) made a significant improvement in producing solutions on Instance 1A as it provides a better %Gap than the exact method. MAAT produced %Gap of 5.71%, an improvement of almost 15% compared to that of the exact method. The use of MAAT also reduced the average portion of the fixed costs to 41.86%, 20.12% lower than that obtained by the exact method. Moreover, MAAT required less than a quarter of the computational time required by CPLEX.

Figure 3 shows the location of the opened facilities for problem P1-I3 ($n = 200$ and $\left| P \right| = 5$) where from 100 potential depots, only 9 need to be opened in order to serve 200 customers. These opened depots will receive 5 types of products from 5 plants and will transfer them to the 200 customers. Here, the demand of a customer for each product is randomly generated between 1 and 5 following a uniform distribution.

Table 1. Computational Results on Instance 1A using the exact method (CPLEX)

Instance	l	m	n	/P/	Exact Method (CPLEX)						
					UB	LB	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P1-I1	5	50	100	5	859,476.96	859,391.60	522,757.00	336,719.96	5	0.01	482
P1-I2	5	50	100	10	1,712,188.94	1,701,654.15	987,330.00	724,858.94	5	0.62	10,943
P1-I3	5	100	200	5	2,144,357.91	2,144,190.15	1,243,286.00	901,071.91	9	0.01	791
P1-I4	5	100	200	10	4,073,169.87	4,028,864.08	2,170,745.00	1,902,424.87	10	1.09	10,860
P1-I5	10	150	300	5	4,126,418.68	3,998,710.75	2,469,046.00	1,657,372.68	11	3.09	10,801
P1-I6	10	150	300	10	7,321,442.84	7,116,346.01	3,825,913.00	3,495,529.84	11	2.80	10,801
P1-I7	10	200	400	5	5,666,516.04	5,592,849.34	2,974,646.00	2,691,870.04	14	1.30	10,801
P1-I8	10	200	400	10	10,903,602.94	10,378,356.86	5,497,694.00	5,405,908.94	18	4.82	10,806
P1-I9	15	250	500	5	7,829,092.39	7,653,136.80	4,318,985.00	3,510,107.39	15	2.25	10,802
P1-I10	15	250	500	10	14,977,757.65	13,817,883.90	7,338,672.00	7,639,085.65	19	7.74	10,824
P1-I11	15	300	600	5	9,999,728.25	9,902,291.61	5,310,636.00	4,689,092.25	18	0.97	10,818
P1-I12	15	300	600	10	20,279,353.91	18,207,948.75	9,690,427.00	10,588,926.91	27	10.21	10,830
P1-I13	20	350	700	5	13,715,668.37	13,009,705.95	7,012,801.00	6,702,867.37	20	5.15	10,822
P1-I14	20	350	700	10	70,181,688.23	22,909,924.38	56,227,713.00	13,953,975.23	126	67.36	10,823
P1-I15	20	400	800	5	24,877,115.19	16,217,960.91	8,680,901.00	16,196,214.19	26	34.81	10,829
P1-I16	20	400	800	10	73,306,428.25	29,114,849.83	54,305,006.00	19,001,422.25	122	60.28	10,899
P1-I17	25	450	900	5	21,937,922.52	20,166,331.34	9,615,279.00	12,322,643.52	23	8.08	11,128
P1-I18	25	450	900	10	182,048,139.23	37,321,943.86	122,778,690.00	59,269,449.23	290	79.50	10,853
P1-I19	25	500	1000	5	36,434,926.44	20,162,054.28	22,961,457.00	13,473,469.44	63	44.66	10,839
P1-I20	25	500	1000	10	162,989,855.27	35,638,668.27	90,669,680.00	72,320,175.27	204	78.13	10,836
Average							61.98%	38.02%		20.64	9,829

Table 2. Computational Results on Instance 1A using the proposed matheuristic (MAAT)

| Instance | *l* | *m* | *n* | \|P\| | Proposed Method (MAAT) | | | | | |
					Z	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P1-I1	5	50	100	5	859,476.96	522,757.00	336,719.96	5	0.01	280
P1-I2	5	50	100	10	1,712,188.94	987,330.00	724,858.94	5	0.62	1,513
P1-I3	5	100	200	5	2,145,765.68	1,238,109.00	907,656.68	9	0.07	158
P1-I4	5	100	200	10	4,037,304.59	2,114,043.00	1,923,261.59	9	0.21	1,426
P1-I5	10	150	300	5	4,109,961.31	2,389,520.00	1,720,441.31	11	2.71	1,566
P1-I6	10	150	300	10	7,382,210.77	3,943,396.00	3,438,814.77	12	3.60	2,274
P1-I7	10	200	400	5	5,643,344.35	2,976,763.00	2,666,581.35	14	0.89	1,842
P1-I8	10	200	400	10	10,641,952.47	4,823,194.00	5,818,758.47	14	2.48	2,551
P1-I9	15	250	500	5	7,825,737.00	4,327,034.00	3,498,703.00	15	2.21	2,148
P1-I10	15	250	500	10	14,140,941.51	6,769,712.00	7,371,229.51	16	2.28	2,863
P1-I11	15	300	600	5	10,001,371.47	5,014,685.00	4,986,686.47	17	0.99	1,181
P1-I12	15	300	600	10	18,629,034.83	7,955,486.00	10,673,548.83	18	2.26	2,773
P1-I13	20	350	700	5	13,560,042.21	6,230,160.00	7,329,882.21	17	4.06	2,387
P1-I14	20	350	700	10	26,480,196.77	11,433,330.00	15,046,866.77	23	13.48	3,365
P1-I15	20	400	800	5	17,366,924.80	7,453,322.00	9,913,602.80	20	6.62	2,524
P1-I16	20	400	800	10	33,701,979.71	12,915,094.00	20,786,885.71	26	13.61	3,617
P1-I17	25	450	900	5	21,680,084.47	8,682,694.00	12,997,390.47	20	6.98	2,678
P1-I18	25	450	900	10	45,050,134.09	15,457,377.00	29,592,757.09	26	17.15	3,776
P1-I19	25	500	1000	5	23,184,691.70	9,987,495.00	13,197,196.70	23	13.04	2,858
P1-I20	25	500	1000	10	45,061,614.02	15,884,255.00	29,177,359.02	26	20.91	4,120
Average						41.86%	58.14%		5.71	2,295
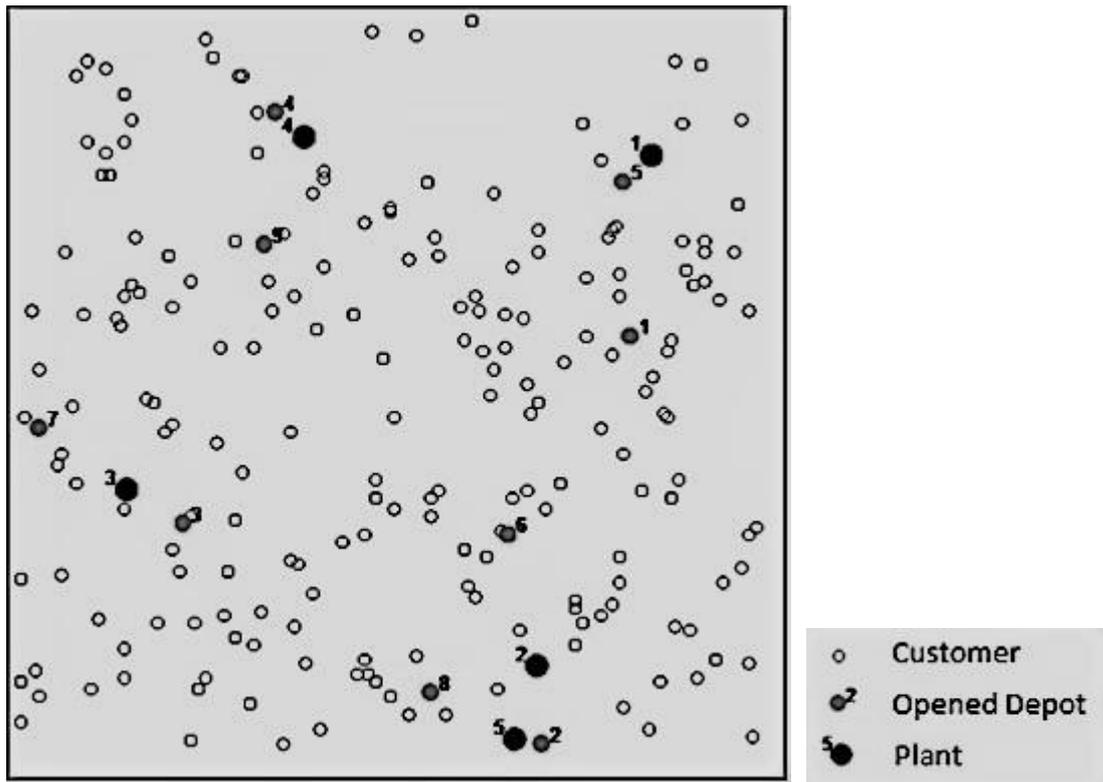
Figure 3. The location of opened depots for problem P1-I3 ($n = 200$ and $|P| = 5$)

Table 3 presents the capacity of each plant for each product (p1 – p5) used in problem P1-I3 ($n = 200$ and $|P| = 5$) where the location of the plants (x and y co-ordinates) is also given. This problem consists of 100 potential depots, each of which has 3 possible capacities to choose from (30, 50, and 70) for each product. In the solution, only 9 depots are selected to be opened in order to minimise the total cost. Their locations and capacity for each product are given in Table 4.

Table 3. The capacity of plants used in problem P1-I3 ($n = 200$ and $|P| = 5$)

Plant	Location		Capacity				
	x	y	p1	p2	p3	p4	p5
1	170	38	132	134	125	136	129
2	139	177	126	132	136	134	129
3	28	129	128	125	124	125	130
4	76	33	126	126	129	131	132
5	133	197	132	121	128	134	132

Table 4. The capacity of depots in the solution for problem P1-I3 (n = 200 and $|P|$ = 5)

Depot	Location		Capacity				
	x	y	p1	p2	p3	p4	p5
1	165	88	70	70	50	70	70
2	141	199	70	70	70	70	70
3	44	139	70	70	70	70	70
4	69	27	70	70	70	70	70
5	163	46	70	70	70	70	70
6	132	142	70	70	70	70	70
7	5	113	50	70	50	50	50
8	111	185	70	70	70	70	70
9	66	63	50	70	70	70	70

Computational Results on Instance 1B

Tables 5 and 6 reveal the computational results on Instance 1B using the exact method and the proposed method (MAAT) respectively. In this instance, CPLEX was not able to solve the problem with $n \geq 600$ and $|P|$ = 10 due to memory issues. Therefore, this instance only consists of 15 problems instead of 20. Using the exact method, without the problems with $n \geq 600$ and $|P|$ = 10, the %Gap value obtained is relatively low as on average, the exact method provided %Gap of 2.82%. The average proportion of fixed costs from the total cost is 38.29% with the total transportation cost contributing the remainder. Similarly to the previous experiments, the proposed matheuristic method (MAAT) performed very well in solving the problems in this instance. The MAAT produced a %Gap of 2.47%, which is better than that obtained by the exact method. Compared to the exact method, the use of the MAAT decreased the average proportion of the fixed cost by 1.6% to 36.69%. Similarly to previous experiments, the MAAT also required less than a quarter of the computational time required by the exact method. In general, based on the computational experiments on Dataset 1, the proposed matheuristic technique (MAAT) runs much faster than the exact method while yielding smaller %Gaps.

Table 5. Computational Results on Instance 1B using the exact method (CPLEX)

Instance	l	m	n	$/P/$	Exact Method (CPLEX)						
					UB	LB	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P2-I1	5	50	100	5	8,705,619.63	8,704,911.45	5,209,800.00	3,495,819.63	3	0.01	157
P2-I2	5	50	100	10	17,002,283.03	17,000,961.88	9,978,990.00	7,023,293.03	5	0.01	118
P2-I3	5	100	200	5	17,621,670.75	17,619,909.10	8,918,790.00	8,702,880.75	5	0.01	815
P2-I4	5	100	200	10	37,125,285.84	37,076,857.88	19,391,400.00	17,733,885.84	10	0.13	10,801
P2-I5	10	150	300	5	29,401,199.80	29,396,265.25	14,113,200.00	15,287,999.80	8	0.02	10,862
P2-I6	10	150	300	10	61,805,645.90	59,680,626.30	28,637,940.00	33,167,705.90	15	3.44	10,801
P2-I7	10	200	400	5	43,257,673.12	42,823,901.32	17,943,030.00	25,314,643.12	10	1.00	10,802
P2-I8	10	200	400	10	92,815,595.76	91,612,669.66	39,252,480.00	53,563,115.76	20	1.30	10,811
P2-I9	15	250	500	5	59,322,626.11	58,633,420.67	24,227,850.00	35,094,776.11	14	1.16	10,802
P2-I10	15	250	500	10	92,610,158.16	91,612,669.66	39,280,980.00	53,329,178.16	20	1.08	10,824
P2-I11	15	300	600	5	85,853,076.49	79,009,642.12	33,168,870.00	52,684,206.49	19	7.97	10,848
P2-I12	20	350	700	5	99,449,406.72	94,600,290.81	35,824,500.00	63,624,906.72	21	4.88	10,810
P2-I13	20	400	800	5	133,408,734.87	121,513,227.81	47,600,130.00	85,808,604.87	27	8.92	10,807
P2-I14	25	450	900	5	150,229,927.89	141,311,588.41	44,080,950.00	106,148,977.89	25	5.94	10,926
P2-I15	25	500	1000	5	161,221,522.14	150,710,307.76	49,644,150.00	111,577,372.14	27	6.52	11,059
Average							38.29%	61.71%		2.82	8,750

Table 6. Computational Results on Instance 1B using the proposed matheuristic (MAAT)

Instance	l	m	n	$|P|$	Proposed Method (MAAT)					
					Z	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P2-I1	5	50	100	5	8,705,619.63	5,209,800.00	3,495,819.63	3	0.01	11
P2-I2	5	50	100	10	17,002,886.93	9,978,990.00	7,023,896.93	5	0.01	95
P2-I3	5	100	200	5	17,621,670.75	8,918,790.00	8,702,880.75	5	0.01	112
P2-I4	5	100	200	10	37,150,129.40	19,392,540.00	17,757,589.40	10	0.20	1,331
P2-I5	10	150	300	5	29,401,746.54	14,113,200.00	15,288,546.54	8	0.02	558
P2-I6	10	150	300	10	60,796,006.69	29,039,220.00	31,756,786.69	17	1.87	3,266
P2-I7	10	200	400	5	43,060,704.37	17,897,430.00	25,163,274.37	10	0.55	2,221
P2-I8	10	200	400	10	95,356,426.66	41,173,950.00	54,182,476.66	22	4.09	3,545
P2-I9	15	250	500	5	60,038,502.75	24,243,810.00	35,794,692.75	14	2.40	2,248
P2-I10	15	250	500	10	95,356,426.66	41,173,950.00	54,182,476.66	22	4.09	3,544
P2-I11	15	300	600	5	81,690,709.57	27,342,900.00	54,347,809.57	15	3.39	3,259
P2-I12	20	350	700	5	96,110,675.18	32,248,890.00	63,861,785.18	19	1.60	3,384
P2-I13	20	400	800	5	129,100,367.37	39,013,080.00	90,087,287.37	22	6.24	3,554
P2-I14	25	450	900	5	151,004,365.49	41,628,810.00	109,375,555.49	24	6.86	3,644
P2-I15	25	500	1000	5	159,230,859.30	45,434,130.00	113,796,729.30	25	5.65	3,824
Average						36.69%	63.31%		2.47	2,306

4.2. Experiments on datasets from the literature and wind energy industry

The performance of our proposed approach is also assessed on an existing dataset taken from literature and those used in the wind energy industry.

Experiments on the existing dataset

We test our proposed method on data sets from Eskandarpour *et al*. (2017) originally used for solving a supply chain network design problem. This existing dataset provides the location of plants, depots and customers where the customer demand for 5 products $(|P| = 5)$ is also given. Here, we use Model A to solve this existing dataset as there is no information related to the dimension of each product. The missing information required to solve Model A is generated based on the total demand for each product. We estimate the capacity of plants and depots along with the fixed cost of opening depot based on its capacity in such way that in a good solution, the total transportation cost is close to the total fixed cost. The existing dataset consists of 15 problems where the number of customer (n) is varied between 60 and 300. Therefore, it can be argued that this existing dataset is relatively small and easier to solve by the exact method than the datasets presented in Section 4.1.

Tables 7 and 8 show the computational results on the existing dataset using the exact method and the proposed method (MAAT) respectively. According to Table 7, CPLEX terminated before the time based termination criterion of 3 hours for 11 of 15 problems where CPLEX stopped because the %Gap between UB and LB has reached the termination level of 0.01%. For the other four problems, the %Gap obtained by CPLEX for these problems is very low after the time based termination criterion of 3 hours, indicating near-optimality. On average, the exact method produced a relatively small %Gap of 0.08% with the total transportation cost contributing approximately 40% of the total cost. The proposed matheuristic method (MAAT) also performs well in this instance. The MAAT yielded a %Gap of 0.1% within a relatively short computational time.

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Table 7. Computational Results on dataset from literature using the exact method (CPLEX)

Instance	l	m	n	$|P|$	Exact Method (CPLEX)						
					UB	LB	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P3-I1	6	12	60	5	55,572,897.41	55,567,375.30	31,694,532.00	23,878,365.41	5	0.01	8
P3-I2	7	14	70	5	69,069,021.29	69,062,114.65	37,191,446.00	31,877,575.29	6	0.01	1,704
P3-I3	8	16	80	5	74,571,746.05	74,564,294.64	40,420,666.00	34,151,080.05	6	0.01	94
P3-I4	9	18	90	5	79,786,050.94	79,778,073.22	44,820,870.00	34,965,180.94	7	0.01	44
P3-I5	10	20	100	5	90,176,144.57	90,167,134.89	51,333,757.00	38,842,387.57	8	0.01	78
P3-I6	12	24	120	5	101,419,657.45	101,409,807.72	60,491,320.00	40,928,337.45	9	0.01	49
P3-I7	14	28	140	5	113,780,685.86	113,769,538.12	68,022,247.00	45,758,438.86	10	0.01	57
P3-I8	16	32	160	5	118,342,803.59	118,330,984.06	66,928,149.00	51,414,654.59	12	0.01	133
P3-I9	18	36	180	5	124,577,109.01	123,981,600.48	77,525,132.00	47,051,977.01	14	0.48	10,809
P3-I10	20	40	200	5	132,750,121.65	132,736,847.40	83,315,233.00	49,434,888.65	15	0.01	6,960
P3-I11	22	44	220	5	144,228,937.54	144,214,536.07	90,863,183.00	53,365,754.54	16	0.01	1,362
P3-I12	24	48	240	5	135,764,873.31	135,664,313.75	82,409,927.00	53,354,946.31	18	0.07	10,863
P3-I13	26	52	260	5	150,178,953.14	150,163,938.36	90,012,918.00	60,166,035.14	19	0.01	2,733
P3-I14	28	56	280	5	151,777,649.65	151,658,391.53	95,612,554.00	56,165,095.65	21	0.08	10,908
P3-I15	30	60	300	5	161,772,569.23	160,907,532.51	101,155,572.00	60,616,997.23	22	0.53	10,864
Average							59.97%	40.03%		0.08	3,778

Table 8. Computational Results on dataset from literature using the proposed matheuristic (MAAT)

Instance	l	m	n	/P/	Proposed Method (MAAT)					
					Z	Opening Cost	Trans Cost	#Opened Warehouse	Gap (%)	CPU (s)
P3-I1	6	12	60	5	55,572,897.41	31,694,532.00	23,878,365.41	5	0.01	10
P3-I2	7	14	70	5	69,069,021.29	37,191,446.00	31,877,575.29	6	0.01	120
P3-I3	8	16	80	5	74,571,746.05	40,420,666.00	34,151,080.05	6	0.01	92
P3-I4	9	18	90	5	79,786,050.94	44,820,870.00	34,965,180.94	7	0.01	50
P3-I5	10	20	100	5	90,176,144.57	51,333,757.00	38,842,387.57	8	0.01	183
P3-I6	12	24	120	5	101,419,657.45	60,491,320.00	40,928,337.45	9	0.01	42
P3-I7	14	28	140	5	113,780,685.86	68,022,247.00	45,758,438.86	10	0.01	100
P3-I8	16	32	160	5	118,347,599.15	66,830,463.00	51,517,136.15	12	0.01	83
P3-I9	18	36	180	5	124,647,837.60	78,115,821.00	46,532,016.60	14	0.54	1,513
P3-I10	20	40	200	5	132,750,705.22	83,320,873.00	49,429,832.22	15	0.01	1,041
P3-I11	22	44	220	5	144,228,937.54	90,863,183.00	53,365,754.54	16	0.01	1,075
P3-I12	24	48	240	5	135,788,271.73	82,440,405.00	53,347,866.73	18	0.09	1,449
P3-I13	26	52	260	5	150,178,953.14	90,012,918.00	60,166,035.14	19	0.01	426
P3-I14	28	56	280	5	151,905,199.85	94,648,871.00	57,256,328.85	21	0.16	1,503
P3-I15	30	60	300	5	161,848,997.48	101,441,406.00	60,407,591.48	22	0.59	1,542
Average						59.95%	40.05%		0.10	615

A. Experiments on dataset from the wind energy industry

Renewable energy sources have attracted a lot of attention in recent years due to several factors including a surge in the world energy demand, limitation of fossil fuel reserves, fossil fuel price instability and global climate change (Abdmouleh *et al*., 2015). The UK Government has set a national target for 15% of its total energy consumption to come from renewable sources by 2020, of which it is expected that wind energy will make the largest single contribution to this target (Jones and Wall, 2016). A wind farm can be located either onshore or offshore. The development of the offshore wind industry has significantly increased over the past 20 years. One of the reasons for this growth is that a wind turbine at sea generally produces more electricity than that of its onshore equivalent as the average wind speed at sea is higher (Irawan *et al*., 2017).

The operations and maintenance (O&M) cost is one of the largest components of the cost of a wind farm. One way to reduce the costs is to make the maintenance activities more efficient by optimising the logistic system in order to reduce turbine downtime. The logistic system should hence be designed to ensure that the spare parts are available and easy to be access when they are needed. In the wind energy sector, spare parts are complex and expensive, characterized by high procurement costs and low inventory levels (Tracht et al., 2013). However, it is critical to manage and maintain an adequate level of spare parts as inadequate stocks when a part fails may stop electricity generation and lead to substantial losses.

Spare parts supplied by plants are delivered to capacitated depots, and then distributed to Operation & Maintenance bases (O&M bases). Depots are usually located near to or at the O&M base locations. However, as the inventory levels of the spare parts are relatively low, depots may not be opened at all O&M bases. This means that a depot may serve more than one O&M base. Moreover, a depot may not store the same parts as other depots. For an example, depot A may store only blades and bearing generators whereas depot B may manage transformers and yaw motors.

Optimization of the location and capacity of maintenance accommodations for offshore wind farms has been investigated by a few researchers. De Regt (2012) studied the optimal location of offshore maintenance accommodations by solving a 'Weber' problem to minimise the weighted sum of distances to given points. Besnard et al. (2013) introduced an optimisation model to find the optimal location of maintenance accommodations, number of technicians,

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choice of transfer vessels and the possibility of using a helicopter to service offshore wind farms.

This section presents computational results of the facility location model for finding the optimal number of depots that need to be opened along with their optimal locations in order to support operation and maintenance of offshore/onshore wind farms in the UK. The capacity to store spare parts for each opened depot is also optimised. Here, Model A is most suitable to be implemented for this case study as the dimension of spare parts and the volume of potential depots are difficult to estimate. In this case, warehouses are treated as depots whereas O&M bases act as customers. The model aims to minimise the total cost which comprises shipment costs (with downtime cost) and capital costs incurred by opening depots. A set of possible product capacities is given where each capacity has a different annual fixed cost which may consist of opening depot and inventory costs. The transfer cost of each component from plants to depots comprises shipping and product costs whereas the one from depots to O&M bases considers downtime and shipping costs. The model will also find the optimal number of depots that need to be opened.

It is common in the wind energy industry that a depot is built to store spare parts of one type of wind turbine. In this case study, the type of wind turbines that we study is Vestas V80/90 as the data for this type of turbine is available in the literature. Therefore, we take into account all wind farm sites (offshore and onshore) in the UK that use the Vestas V80/90 wind turbine. Table 9 shows the detailed wind farm data of sites in the UK that use the Vestas V80/90 (www.renewableuk.com) including the West Gabbard offshore site which is currently still under construction. The table presents the location of wind farms along with number of turbines and installed capacity. Moreover, the table also reveals the location of the associated O&M base for each wind farm. These O&M base locations are also treated as potential depot locations. Table 10 shows the detailed information on parts considered in the case study where the cost and failure rate of each part are given. The part information is based on Lindqvist and Lundin (2010). We also assume that all spare parts are supplied by the manufacturer Vestas located in Randers, Denmark whose coordinates are (Lat 56.433127, Lon 10.047057). In other words, the number of plants is set to one ($l = 1$).

Table 9. Wind farms in the UK that use Vestas V80/90

Initial	Name	Region	Latitude	Longitude	Installed Capacity (MW)	Number of Turbines	Turbine Model	O&M base		
								Name	Latitude	Longitude
Offshore wind farm										
Off1	North Hoyle	North Wales	53.4178	-3.4478	60	30	Vestas V80-2.0 MW	Mostyn	53.321279	-3.262994
Off2	Scroby Sands	Norfolk	52.6458	1.7876	60	30	Vestas V80-2.0 MW	Great Yarmouth	52.592932	1.727134
Off3	Barrow	Cumbria	53.9875	-3.2702	90	30	Vestas V90-3.0 MW	Barrow	54.098699	-3.223713
Off4	Kentish Flats 1	Kent	51.4616	1.0933	90	30	Vestas V90-3.0 MW	Whitstable	51.362906	1.027905
Off5	Robin Rigg	Cumbria	54.7465	-3.6925	180	60	Vestas V90-3.0 MW	Workington	54.649001	-3.565064
Off6	Thanet	Kent	51.4306	1.6331	300	100	Vestas V90-3.0 MW	Ramsgate	51.3333	1.41667
Off7	West Gabbard	Suffolk	51.98	2.08	375	125	Vestas V90-3.0 MW	Lowestoft	52.4833	1.75
Onshore wind farm (O&M base is located in wind farm site)										
On1	Stags Holt	Cambridgeshire	52.57472	-0.14583	18	9	Vestas V80-2.0 MW			
On2	Goonhilly Repowering	Cornwall	50.04611	-5.19889	12	6	Vestas V80-2.0 MW			
On3	Wolf Bog	Co Antrim	54.80306	-6.09417	10	5	Vestas V80-2.0 MW			
On4	North Rhins	Dumfries & Galloway	54.88194	-5.08333	22	11	Vestas V80-2.0 MW			
On5	Ardrossan (with Extension)	North Ayrshire	55.68583	-4.80722	30	15	Vestas V80-2.0 MW			
On6	Braes of Doune	Stirling	56.27611	-4.0625	72	36	Vestas V80-2.0 MW			
On7	Pates Hill	West Lothian	55.80889	-3.59917	14	7	Vestas V80-2.0 MW			
On8	Milton Keynes	Buckinghamshire	52.13611	-0.66444	14	7	Vestas V90-2.0 MW			
On9	McCain Foods	Cambridgeshire	52.56111	-0.17222	9	3	Vestas V90-3.0 MW			
On10	North Pickenham	Norfolk	52.62611	0.74972	14.4	8	Vestas V90-1.8 MW			
On11	Lindhurst	Nottinghamshire	53.11611	-1.14667	9	5	Vestas V90-1.8 MW			
On12	Garves Mountain/Dunloy	Antrim	55.26611	-6.443222	15	5	Vestas V90-3.0 MW			
On13	Slieve Rushen Repowering	Co Fermanagh	54.16	-7.62	54	18	Vestas V90-3.0 MW			
On14	Aikengall	East Lothian	55.92667	-2.45778	48	16	Vestas V90-3.0 MW			
On15	Wardlaw Wood	North Ayrshire	55.71056	-4.72333	18	6	Vestas V90-3.0 MW			

Table 10. Parts specification

Initial	Spare part	Price (euro)	Failure rate	Depot possible capacities			Initial	Spare part	Price (euro)	Failure rate	Depot possible capacities		
				1	2	3					1	2	3
S1	Blade	75000	0.55	1	2	3	S19	SKIIP 2	1800	1.73	3	6	9
S2	Proportional valve	1800	5.48	9	18	27	S20	EMC filter	1800	5.48	9	18	27
S3	Piston accumulator	1800	5.48	9	18	27	S21	Capacitators	200	17.32	29	58	86
S4	Encoder	600	1.73	3	6	9	S22	CT 3220 FFFF	600	1.73	3	6	9
S5	Bearing generator	1800	0.55	1	2	3	S23	CT 316 VCMS	1800	0.55	1	2	3
S6	Generator fan 1	600	0.55	1	2	3	S24	CT 3601	1800	0.55	1	2	3
S7	Generator fan 2	600	1.73	3	6	9	S25	CT 3133	600	17.32	29	58	86
S8	Encoder rotor	200	17.32	29	58	86	S26	CT 3220 FFFC	1800	1.73	3	6	9
S9	Slip ring fan	1800	5.48	9	18	27	S27	CT 3218	200	1.73	3	6	9
S10	Fan	600	0.55	1	2	3	S28	CT 3614	600	1.73	3	6	9
S11	Motor for cooling system	600	5.48	9	18	27	S29	CT 3363	600	1.73	3	6	9
S12	Yaw gear (right)	1800	1.73	3	6	9	S30	CT 3153	600	5.48	9	18	27
S13	Yaw gear (left)	1800	1.73	3	6	9	S31	CT 279 VOG	200	5.48	9	18	27
S14	Yaw motor	1800	0.55	1	2	3	S32	Transformer	42000	1.73	3	6	9
S15	Mechanic gear for oil pump	1800	1.73	3	6	9	S33	Phase compensator generator	600	17.32	29	58	86
S16	Chopper module	1800	1.73	3	6	9	S34	Q8 main switch	3600	17.32	29	58	86
S17	TRU card	600	1.73	3	6	9	S35	Q8 electric gear	1800	5.48	9	18	27
S18	SKIIP 1	1800	17.32	29	58	86	S36	Q8 EMC filter	200	17.32	29	58	86

Failure rate: per 10^6 hours of operation

A wind farm, consisting of several wind turbines with the same specification, is treated as a customer. A wind turbine is formed of several parts which are considered as products. The average frequency of failure of the part per year is defined as failure rate. Therefore, the demand of each part per year for a windfarm is then based on the number of turbines in the windfarm and its failure rate. In other words, the demand of each spare part per year for each wind farm is calculated as the product of the failure rate value and the number of turbines installed in a wind farm. This is acceptable as the number of spare parts needed is determined by how often the part breaks down. Table 10 also presents the possible depot capacities for each spare part, generated based on the demand of each product. The model will select the best capacity for each opened depot.

In the experiments, we also assume that the holding cost per year of each spare part is 20% of its cost. This information is used to calculate the fixed cost. The transportation cost for each component is based on the distance and we set the maximum transportation cost to 20% of the component cost. To implement Model A on this wind energy case study, minor revisions of the mathematical model are needed. First, variable X_{jkp} is treated as a real value instead of an integer as the demand of products for each O&M base is calculated based on failure rates. Second, the equalities on Constraints (6) are replaced by inequalities ($\geq$). This dataset can be solved optimally using the exact method (CPLEX) within a relatively short computational time as the problem is relatively small ($l = 1$, $m = 22$, $n = 22$, and $|P| = 36$). Therefore, the matheuristic approach (MAAT) is not required to solve this instance.

The optimal solution for this problem reveals that only 4 depots are required to open in order to store the spare parts. Three depots are located on the coast, namely Great Yarmouth, Workington and Ramsgate whereas another depot is located at the Braes of Doune inland windfarm site. This solution is acceptable as there are more offshore than onshore wind turbines. The other main advantage of locating a depot at port is its accessibility from the supplier and customer (O&M base). Moreover, the inland depot is located at the onshore windfarm site that has the largest number of wind turbines. It can also be noted that the locations of the depots are scattered across the UK. The objective function value (the total cost) obtained is 11,451,764.05 where the fixed (opening) cost contributes approximately 35% of the total cost. Table 11 shows the depot configuration located in Port Great Yarmouth, which stores all types of spare parts in the optimal solution.

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Table 11. The depot configuration for Port Great Yarmouth

Spare part		Capacity	Spare part		Capacity
S1	Blade	3	S19	SKIIP 2	3
S2	Proportional valve	9	S20	EMC filter	9
S3	Piston accumulator	9	S21	Capacitators	29
S4	Encoder	3	S22	CT 3220 FFFF	3
S5	Bearing generator	1	S23	CT 316 VCMS	1
S6	Generator fan 1	1	S24	CT 3601	1
S7	Generator fan 2	3	S25	CT 3133	29
S8	Encoder rotor	29	S26	CT 3220 FFFC	3
S9	Slip ring fan	9	S27	CT 3218	3
S10	Fan	1	S28	CT 3614	3
S11	Motor for cooling system	9	S29	CT 3363	3
S12	Yaw gear (right)	3	S30	CT 3153	9
S13	Yaw gear (left)	3	S31	CT 279 VOG	9
S14	Yaw motor	1	S32	Transformer	9
S15	Mechanic gear for oil pump	3	S33	Phase compensator generator	29
S16	Chopper module	3	S34	Q8 main switch	29
S17	TRU card	3	S35	Q8 electric gear	9
S18	SKIIP 1	29	S36	Q8 EMC filter	29

5. Conclusion and suggestions

This paper studies the two-stage capacitated facility location problem with multilevel capacities where the problem is to find the optimal number of depots that need to be opened along with their optimal location and corresponding capacity. We proposed two integer linear programming (ILP) models to address the problem in order to minimise the fixed cost of opening depots and transportation costs. The first model considers the capacity based on the maximum number of products that can be stored whereas in the second one, the capacity is based on the size (volume) of the depot. As large problems are very hard to solve using an exact method, a matheuristic approach, MAAT (Matheuristic Approach incorporating an Aggregation Technique), is introduced to overcome this weakness. The proposed method is evaluated using a randomly generated dataset and datasets taken from literature and the wind energy sector in the UK. According to the computational experiments, the proposed methods ran efficiently, producing a small %Gap within a short computational time.

The models developed in Section 2 can be implemented not only for the wind power sector but also for other industries that need depots to support their business. The models can be enhanced to become bi-objective as there is an underlying trade-off between minimising the number of opened depots and minimising the total costs. The compromise programming method (see Irawan *et al*. (2015) for more detailed information) can be applied to address the trade-off that occurs. The models can also be extended by considering uncertain customer demand. In this case, a technique such as the stochastic programming could be implemented in order to model the problem.

Acknowledgments

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