Information Quality Driven Business Cycles*

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Abstract

This paper introduces information quality in a real business cycle model. Information quality relates to the idea that information obtained can inaccurately reflect the actual state of the economy. Using the Survey of Professional Forecasters, I document that forecast errors are larger during downturns, even if agents acquire more information. I then augment a rational inattention model with information search frictions that generate variable information quality. Information depends on both data abundance and information search intensity. Unlike rational inattention models, which are demand driven, I allow for time-varying data abundance, or information supply, generating fluctuations in information quality. The model delivers pro-cyclical information quality, which rationalizes puzzling evidence that information acquisition and uncertainty increase in downturns. A Bayesian estimation of the model for the US economy shows that information quality accounts for sizable fluctuations in uncertainty and output. The model also generates: (i) systematic mistakes when agents do not internalize fluctuations in information quality, (ii) variation in information processing costs, which produce higher frequency and dispersion in price changes during downturns, and (iii) production externalities, as firms do not internalize that more activity generates data abundance, which reduces uncertainty.

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1 Introduction

Models with incomplete information or information frictions represent a source of macroeconomic fluctuations and propagation (e.g., Nimark 2014 and Maćkowiak and Wiederholt 2015). The presence of information frictions is supported by evidence from surveys on expectations (e.g., Coibion and Gorodnichenko 2012 and Coibion and Gorodnichenko 2015). Most existing models can endogenize information frictions by considering a framework in which the amount of information acquired is determined by the decision makers' demand for information (e.g., Reis 2006, Woodford 2009).[1] While full information is available in principle, agents are subject to information acquisition costs when they choose how much information to acquire. In these frameworks, information acquired varies over time and depends on incentives that respond to business cycles.

In this paper, I begin by observing that information acquisition models inherently equate information acquisition to uncertainty reduction. However, empirical evidence shows that higher measured information acquisition about the aggregate state coincides with periods of higher macroeconomic uncertainty. In fact, information acquisition is counter-cyclical (e.g., Chiang 2021, Flynn and Sastry 2021), while uncertainty also increases in downturns (e.g., Bloom 2009 and Bloom et al. 2018). This constitutes a puzzle for models of information acquisition.

To address this puzzle, this paper introduces cyclical fluctuations in information quality. The key idea is that even though information acquisition increases in a downturn, the quality of information acquired is lower, and hence, forecasts are inaccurate, and uncertainty remains high. Imperfect information quality relates to the idea that information obtained can contain inaccuracies about the state of the economy. Using the Survey of Professional Forecasters, I extend the noisy information framework of Coibion and Gorodnichenko (2015) and find that even after accounting for a rise in information acquisition, average expectational errors are higher in recessions, which indicates lower information quality in downturns.[2]

Motivated by the empirical evidence, I introduce information search frictions in an otherwise standard rational inattention model to generate variable information quality.[3] I use mutual information in Shannon (1948), which is the reduction in

[1]Information frictions can also be modeled with an exogenous signal structure (e.g., Lucas 1972, Woodford 2002). However, it has been documented that information acquisition varies over the cycle and responds to incentives. Hence, these models cannot explain these stylized facts.

[2]For implications of noisy information models on business cycle dynamics, see Lorenzoni (2009), Angeletos and La'O (2010), Angeletos and La'O (2012), Ordonez (2013), Hassan (2014) and Hassan (2017).

[3]Some applications of rational inattention include Lou (2008), Paciello and Wiederholt (2014) Acharya and Wee (2020), Afrouzi (2020), Gabaix (2020) and Morales-Jimenez and Stevens (2022).

uncertainty, as a factor of production. Information search frictions imply that mutual information depends on two forces in general equilibrium: the supply of information and information search intensity (or the demand for information). Unlike rational inattention models, which are demand-driven and assume perfect and constant information supply, I allow information supply to vary with economic conditions.

I model information supply as a by-product of economic activity (e.g., Farboodi and Veldkamp 2019). During a boom, higher activity generates more data points and hence, more information supply. A higher abundance of data generates more mutual information and less economic uncertainty. Information quality then depends on fluctuations in information supply, a channel which is absent in rational inattention models. The demand for mutual information depends on information search intensity. When firms search for more information, this leads to higher mutual information, all else equal.

I show how this framework can rationalize the joint counter-cyclical behavior of information acquisition and uncertainty.[4] Consider an adverse shock that decreases output. This reduces the supply of information due to lower economic activity, which implies lower information quality. As such, the quantity of mutual information decreases. The decline in mutual information leads to a rise in the marginal benefit of searching for information, increasing the incentive to search for information. Hence, economic agents search more for information in a downturn.

The contrasting effects of the decline in information supply and the increase in search intensity lead to ambiguous effects on the amount of information in the economy. When I discipline the parameters of the model with data using Bayesian estimation, I find that the net result is lower information in a downturn, indicating higher uncertainty. Hence, the model proposes an empirical resolution of the joint counter-cyclical behavior of information acquisition and uncertainty.

Next, I examine the quantitative implications of information quality. I demonstrate that the amplification and persistence of downturns in standard rational inattention models rely on pro-cyclical information acquisition. Consider an adverse productivity shock, which decreases expected profits. Pro-cyclical information acquisition in rational inattention models implies a decline in mutual information and a rise in uncertainty, which further depresses output and expected profits. This creates an amplification loop between output and uncertainty, which explains severe downturns.

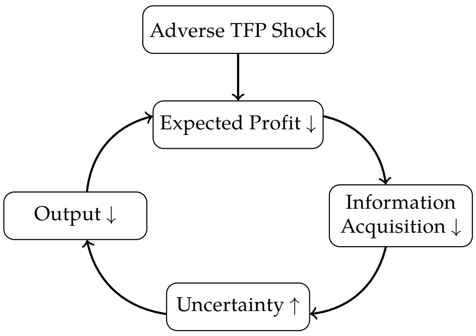
However, pro-cyclical information acquisition is inconsistent with empirical ev-

[4] In principle, information acquisition can increase in downturns because the downturn results from exogenous uncertainty shocks. However, my framework considers endogenous fluctuations in uncertainty. In addition, Ludvigson et al. (2021) provides empirical evidence that macroeconomic uncertainty tends to be an endogenous response to other shocks.

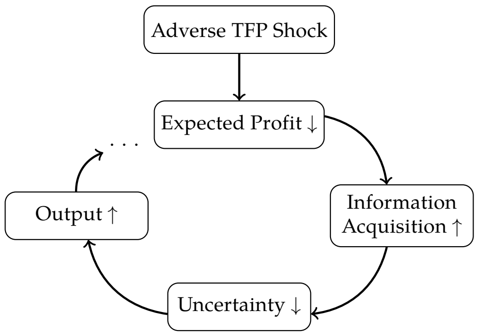
idence. If one specifies a standard rational inattention model to generate counter-cyclical information acquisition, this leads to a dampening of recessions. Suppose information acquisition rises when output and expected profits decline due to an adverse shock. Counter-cyclical information acquisition increases mutual information and decreases uncertainty. This leads to an increase in output and dampens the effect of the adverse shock. Hence, counter-cyclical information acquisition cannot be a source of business cycle amplification.

By introducing information quality and information search frictions in a rational inattention framework, I can generate amplification and counter-cyclical information acquisition simultaneously. During a downturn, the increase in information search intensity tends to reduce uncertainty. However, as mutual information also depends on fluctuations in information supply, information scarcity implies that information quality declines, which increases uncertainty. The net effect in my calibrated model is a rise in uncertainty, which restores the amplification loop between output and uncertainty.

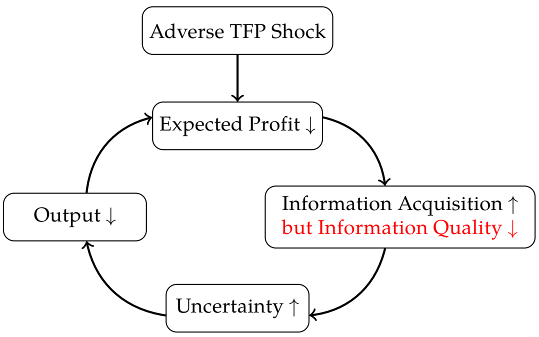
Figure 1: Business Cycle Dynamics



(a) Pro-cyclical Information Acquisition



(b) Counter-cyclical Information Acquisition



(c) Counter-cyclical Information Acquisition with Information Quality

Notes: This figure presents the mechanisms behind different models of information acquisition.

Furthermore, I demonstrate that information search frictions are essential in generating severe downturns in my model. The introduction of information search frictions, which formalizes information as a joint product of general equilibrium forces, generates larger imperfections of information quality, which leads to stronger increases in uncertainty and declines in output during downturns.

Quantitatively, fluctuations in information quality create substantial changes in uncertainty, explaining approximately 24 percent of fluctuations in output. In addition, cyclical variations in information quality are largely attributed to information search frictions.

Lastly, I also consider several extensions of the model. In the rational inattention model with information search frictions, economic agents internalize that information quality is time-varying. When agents do not internalize fluctuations in information quality, they make systematic mistakes. I find that systematic mistakes generate bigger losses in output during recessions. Moreover, the cost of mistakes increases as the elasticity of substitution increases. The model also implies time-varying information processing costs. I show that information processing costs tend to be larger in downturns, which implies larger dispersion in price changes. Appropriate policies can correct production externalities in the model, in which firms do not internalize that producing more output can benefit others by generating more data.

Related Literature. This study contributes to three strands of the literature. First, my work is related to the empirical evidence of counter-cyclical information acquisition. Flynn and Sastry (2021) use data from US public firms' regulatory filings and financial statements to document that the firms' attention to macroeconomic conditions increases during downturns. Chiang (2021) uses Google traffic data to document higher search intensity in recessions. Song and Stern (2021) use a text-based approach to provide evidence of counter-cyclical attention. Macaulay (2022) uses data on the retail savings market to find that households allocate counter-cyclical attention to their savings.

My work also relates to models that generate counter-cyclical information acquisition by relying on different mechanisms. Flynn and Sastry (2021) and Macaulay (2022) assume that risk-averse agents pay more attention to macroeconomic conditions when aggregate consumption is low and the marginal utility of consumption is high. Mäkinen and Ohl (2015) demonstrate that price fluctuations affect the incentive to acquire information by learning from prices. Chiang (2021) considers a strategic approach to acquiring information, in which reacting more to an event generates more volatility, and each agent faces more uncertainty regarding the aggregate actions of others. My paper generates counter-cyclical information acquisition through lower information quality in a downturn.

Counter-cyclical information acquisition by itself tends to generate pro-cyclical uncertainty, in which uncertainty is defined as the *perceived* volatility of shocks (e.g., Jurado et al. 2015).[5] Pro-cyclical uncertainty, in turn, leads to the dampening of shocks. By contrast, the amplification of downturns relies on counter-cyclical uncertainty. Hence, counter-cyclical information acquisition cannot explain deep and long-lasting recessions through the uncertainty channel. I contribute to this literature by providing a model that generates both counter-cyclical information acquisition and counter-cyclical perceived uncertainty. Thus, my model can match empirical facts on cyclical information acquisition and amplify recessions at the same time.

Second, my work relates to the literature that generates pro-cyclical information acquisition. The amplification and persistence of downturns in time-varying uncertainty models rely on pro-cyclical information acquisition. In Van Nieuwerburgh and Veldkamp (2006) and Saijo (2017), the noise-to-signal ratio is counter-cyclical so that learning and information acquisition are pro-cyclical, which generates asymmetric and severe recessions. Fajgelbaum et al. (2017) find that firms acquire more information by investing more in good times. Although their mechanism relies on "learning by doing", the impacts are analogous to pro-cyclical information acquisition, which is inconsistent with empirical evidence. In addition, one can argue that the cyclicality of information acquisition does not matter as long as one can generate counter-cyclical uncertainty to explain severe recessions. By embedding information search frictions in a rational inattention model, I generate even stronger increases in uncertainty during downturns than models with pro-cyclical information acquisition.[6]

Third, this paper is related to the literature on information rigidity and its macroeconomic implications. Coibion and Gorodnichenko (2015) examine consensus forecasts and document information rigidities relative to the full information rational expectations (FIRE) benchmark.[7] I extend the noisy information framework of Coibion and Gorodnichenko (2015) to construct a measure of information quality. After accounting for information rigidity, I include a wedge in the noisy information model that absorbs average expectational errors across different aggregates and forecast horizons, which proxies for information quality. This gives rise to alternative interpretations of the regressions in Coibion and Gorodnichenko (2015).

Layout. The remainder of the paper is organized as follows. Section 2 documents

[5]Other definitions of uncertainty include actual volatility. For models with endogenous volatility, see Bachmann and Moscarini (2011), Fostel and Geanakoplos (2012), Ilut et al. (2018) and Bernstein et al. (2022).

[6]For applications of uncertainty shocks, see Fernández-Villaverde et al. (2011), Mumtaz and Zanetti (2013), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017), Arellano et al. (2019) and Fernández-Villaverde and Guerrón-Quintana (2016).

[7]Other studies or tests of the FIRE benchmark include Andrade and Le Bihan (2013), Coibion et al. (2018) and Coibion et al. (2020).

empirical evidence of lower information quality during downturns. Section 3 outlines a simple model of information quality. Section 4 embeds this simple model in a quantitative New Keynesian model; explains the parametrization, calibration, and estimation strategy; and presents the results. Section 5 studies the quantitative implications of time-varying information quality. Section 6 concludes.

2 Empirical Evidence

I extend the empirical framework in Coibion and Gorodnichenko (2015) to test for time-varying information quality. I provide indirect empirical evidence that information quality declines in a downturn.

2.1 Data

My analysis requires data on expectations. I use the Survey of Professional Forecasters (SPF), provided by the Federal Reserve Bank of Philadelphia. The survey is conducted on a sample of 40 professional forecasters in each quarter. The data includes individual forecasts for the current and subsequent four quarters for several macroeconomic outcomes such as GDP, price indices, consumption, investment, and unemployment.

2.2 Methodology

I follow Coibion and Gorodnichenko (2015) and base my empirical tests on the following noisy information model.[8]

A: Noisy Information Model in Coibion and Gorodnichenko (2015)

In this model, agents continuously update their information sets but never acquire full information about the state. This is a signal extraction problem, in which agents receive a signal $s_{i,t}^{z,A}$ of a hidden state z_t, where

$$s_{i,t}^{z,A} = z_t + v_{i,t} \tag{1}$$

where $v_{i,t}$ is a random variable normally distributed with mean zero and i.i.d across time and agents. We can measure information rigidity as the variance of $v_{i,t}$.

[8] The model in this paper can also be interpreted as a sticky information model, as in Coibion and Gorodnichenko (2015). See Appendix for more details.

Each agent i uses the Kalman Filter to generate forecasts of z_t conditional of observing the signal $s_{i,t}^{z,A}$

$$F_{i,t}z_t = Gs_{i,t}^{z,A} + (1-G)F_{i,t-1}z_t \tag{2}$$

where $F_{i,t}$ is the forecast of z_t of agent i at time t and G is the Kalman gain, which represents the relative weight of the informativeness of the signal $s_{i,t}^{z}$, as compared to past information. The Kalman gain depends on the noise-to-signal ratio, which is the ratio of the variances of $v_{i,t}$ to z_t. A higher degree of information rigidity (or higher variance of $v_{i,t}$) translates into a lower value of G. This is because the signal exhibits relatively more noise, and hence when forming posterior expectations, agents rely less on the signal by having a lower Kalman gain. Coibion and Gorodnichenko (2015) average Eq. (2) across agents to arrive at

$$z_{j,t+h} - F_t z_{j,t+h} = \underbrace{\frac{1-G}{G}}_{\beta_1}(F_t z_{j,t+h} - F_{t-1}z_{j,t+h}) + v_{j,t+h,t}^{A} \tag{3}$$

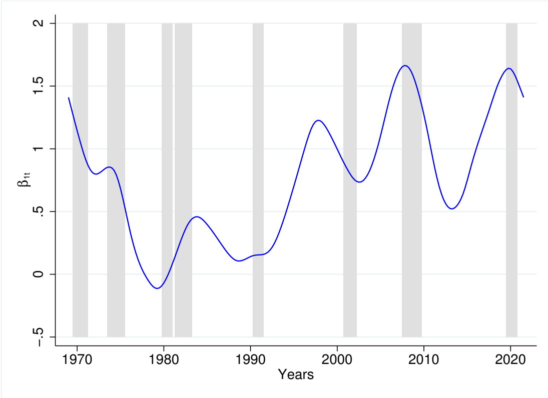
where $z_{j,t+h} - F_t z_{j,t+h}$ denote forecast errors, which measure the difference between the realization and the forecast of $z_{j,t+h}$ at time t for macroeconomic variable j; $F_t z_{j,t+h} - F_{t-1}z_{j,t+h}$ denote forecast revisions, which measure how forecasters update their forecasts between $t-1$ and t for macroeconomic variable j; and $v_{j,t+h,t}^{A}$ is an expectational error, which denotes the component of forecast errors that cannot be explained by forecast revisions. Under full information rational expectations, β_1 should be equal to zero, as forecast errors should be unpredictable. However, Coibion and Gorodnichenko (2015) find evidence of information rigidity, in which the Kalman gain G is less than one and β_1 is greater than zero.

Moreover, they run the regression in Eq. (3) across various macroeconomic variables j in each quarter t and extract time-varying coefficients $\beta_{1,t}$. They find that their measure of information rigidity declines in downturns. This implies that forecasters update their information sets more in recessions, which points to counter-cyclical information acquisition.

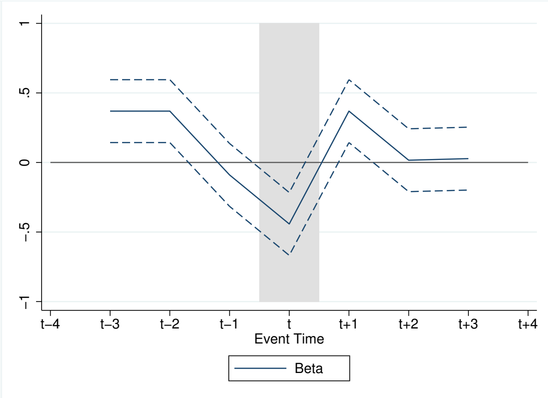
The left panel in Figure 2 plots their measure of information rigidity over time and shows that information rigidity tends to decline in recessions (shaded in gray). I also consider an event study exercise to illustrate the decline in information rigidity. I denote the time period $t-i$ as i periods before a recession at time t and $t+i$ as i periods after a recession. The right panel in Figure 2 shows that $\beta_{1,t}$ tends to decrease in recessions, which indicates that information rigidity declines in downturns.

B: Noisy Information Model with Information Quality

Figure 2: Plot of Information Rigidity Measure



(a) $\beta_{1,t}$ Plot Over Time



(b) Event Study of $\beta_{1,t}$

Notes: This figure presents dynamics of the measure of information rigidity β_1. Panel (a) plots changes in the measure from 1969Q4 to 2020Q2. The measure of information rigidity is smoothed using a Hodrick-Prescott filter. NBER recessions are shaded in gray. Panel (b) shows $\Delta\beta_{1,t}$ using an event study approach. Dotted lines denote 95 % standard error bands.

Next, I show that even though forecasters receive more information in a downturn, forecast errors increase simultaneously, suggesting lower information quality. To extract measures of information quality, I use the following signal structure:

$$s_{i,t}^{z,B} = \chi_t^{\text{emp}} z_t + v_{i,t} \tag{4}$$

I account for information quality in Model B by introducing a wedge χ_t^{emp}.[9] Essentially, χ_t^{emp} represents the average expectational error in Eq. (3), conditional on forecast revisions and the signal structure. If information quality is perfect, then the information source produces signals (conditional on individual noise $v_{i,t}$) that are an unbiased forecast of the true hidden state, and correspondingly expectational errors equal zero. In this case, χ_t^{emp} equals one. When information quality is low, the source of information produces signals (conditional on individual noise $v_{i,t}$) that are a biased forecast of the true hidden state. Correspondingly, expectational errors are large in absolute terms. Hence, lower information quality generates a wedge χ_t^{emp} that deviates away from one.[10] Therefore, the wedge χ_t^{emp} accounts for expectational errors due to low information quality.

Each agent i then uses the Kalman Filter to generate forecasts of z_t conditional on observing the signal $s_{i,t}^{z,B}$:

[9] I also consider an alternative scenario with an additive wedge instead of a multiplicative wedge. See the Appendix for more details.

[10] Model B implies that expectational errors are equal to zero on average across various macroeconomic variables. However, expectational errors are still present for each macroeconomic variable.

$$F_{i,t}z_t = Gs_{i,t}^{z,B} + (1-G)F_{i,t-1}z_t \tag{5}$$

This leads to the following reduced-form regression

$$z_{j,t+h} - F_t z_{j,t+h} = \underbrace{\frac{1-G\chi^{\text{emp}}}{G\chi^{\text{emp}}}}_{\beta_2} F_t z_{j,t+h} - \underbrace{\frac{1-G}{G\chi^{\text{emp}}}}_{\beta_3} F_{t-1} z_{j,t+h} + v_{j,t+h,t}^B \tag{6}$$

In Model A, Coibion and Gorodnichenko (2015) impose the restriction that $\beta_2 = \beta_3$. In Model B, I allow β_2 to be different from β_3 to extract a measure of information quality. In other words, testing for imperfect information quality amounts to a test of the difference between β_2 and β_3. Coibion and Gorodnichenko (2015) find that β_2 is not statistically different from β_3, which implies that χ^{emp} averages to one across time.

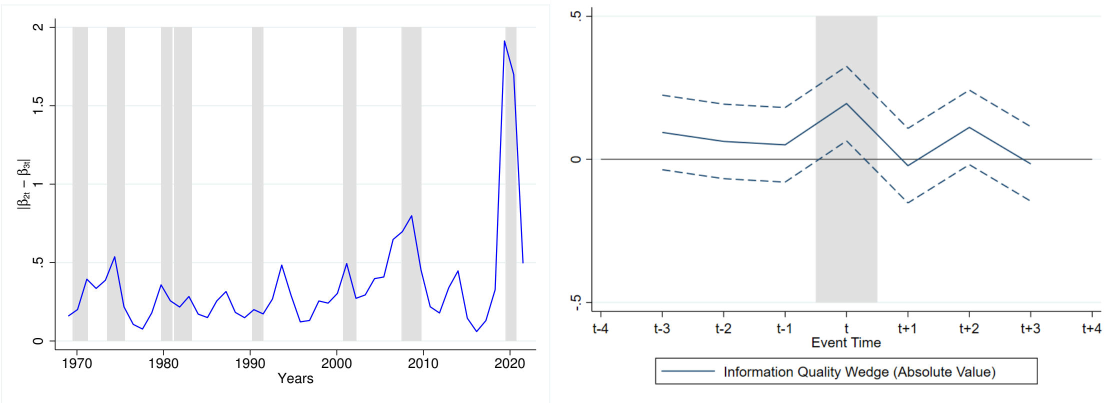
To test for time-varying information quality, I run the regression in Eq. (6) across various macroeconomic variables j in each quarter t. Since there are two unknowns G_t and χ_t^{emp}, and two corresponding coefficients $\beta_{2,t}$ and $\beta_{3,t}$, Eq. (6) is exactly identified. I consider the following measure of information quality:

$$|\beta_{2,t} - \beta_{3,t}| = |\frac{1}{\chi_t^{\text{emp}}} - 1| \tag{7}$$

Low information quality and large expectational errors conditional on the signals received imply that $\beta_{2.t}$ deviates away from $\beta_{3.t}$, as χ_t^{emp} deviates from one.

Figure 3 Panel (a) plots the relevant measure of information quality over time. Even after accounting for lower information rigidities in downturns, which translate to higher information acquisition, the magnitudes of deviations of β_2 from β_3 tend to be higher in recessions (shaded gray areas, dated by NBER), which imply larger expectational errors. I also employ an event study approach to plot the measure of information quality. I denote t as the recession period, $t-i$ as i quarters before a recession, and $t+i$ as i quarters after a recession. Figure 3 Panel (b) shows a significant increase in the information quality measure $|\beta_{2,t} - \beta_{3,t}|$ during a recession. This implies that information quality declines in downturns. Motivated by this evidence, I next build a theoretical model in which both information quality and information acquisition can vary over time.

Figure 3: Plot of Information Quality Measure



(a) $|\beta_{2,t} - \beta_{3,t}|$ Plot Over Time

(b) Event Study of $|\beta_{2,t} - \beta_{3,t}|$

Notes:This figure presents the measure of information quality $|\beta_{2,t} - \beta_{3,t}|$. The measure of information quality is constructed using average expectational errors across 15 different macroeconomic aggregates and horizons $h = 1, 2, 3$. Panel (a) plots the measure from 1969Q4 to 2020Q2. The measure of information rigidity is smoothed using a local average which uses Epanechnikov kernel with bandwidth equal to 0.5. Shaded in gray are the NBER recession dates. Panel (b) shows the measure of information quality using an event study approach. Dotted lines denote 95 % standard error bands.

3 Model

I build a partial equilibrium static information acquisition model with two key ingredients: rational inattention and information search frictions. I first consider a model of rational inattention without information search frictions and show how its implications are inconsistent with the evidence on information rigidity presented in Coibion and Gorodnichenko (2015). Next, I introduce information search frictions and information quality in the model to illustrate the mechanisms that can explain counter-cyclical behavior of both information acquisition behavior and uncertainty. I denote this model as the model with search frictions. Lastly, I compare both models and demonstrate why it is necessary to introduce information search frictions, even though one can in principle obtain similar amplified output and uncertainty dynamics with pro-cyclical information acquisition in the rational inattention model.

3.1 Agents

Representative Household. The household maximizes utility, consisting of consumption and labor:

$$\max_{C_t, N_t} \log C_t - \phi \frac{N_t^{1+\eta}}{1+\eta}$$

subject to

$$W_t N_t = C_t$$

The consumer supplies labor N_t to firms at wage rate W_t. There is no saving, so the consumer chooses consumption C_t in each period to maximize current utility. The first-order condition for the household's maximization problem is given by:

$$\frac{W_t}{C_t} = \phi N_t^{\eta} \tag{8}$$

Final Goods Producers. There are competitive firms that produce final goods under perfect information, with the aggregate production function:

$$Y_t = \Big[\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\Big]^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

where $\epsilon > 1$ is the elasticity of substitution.

The profit maximizing input choice satisfies

$$Y_{i,t} = P_{i,t}^{-\epsilon} Y_t \tag{10}$$

Intermediate Goods Producers. There is a continuum of firms with measure one that produce intermediate goods. Each firm i is a monopolist of good i with production function:

$$Y_{i,t} = Z_t N_{i,t} \tag{11}$$

Firm i produces $Y_{i,t}$ to maximize its profit under uncertainty about aggregate productivity Z_t. Denote

$$z_t = \log Z_t \tag{12}$$

where $z_t \sim N(0, \frac{1}{\tau_z})$ is an aggregate shock that is common to all firms. Firms cannot observe z_t. Instead, each firm i observes a signal of z_t, given by:

$$s_{i,t}^z = z_t + v_{i,t} \tag{13}$$

where the signal contains the actual realization of z_t and noise $v_{i,t} \sim N(0, \frac{1}{\tau_{v,i,t}})$. Denote uncertainty as $Var(z_t | s_{i,t}^z)$, which is the perceived variance of z_t after observing the signal $s^z_{i,t}$. The presence of noise leads to uncertainty. Each firm can acquire information at a cost by increasing the precision of the signal $\tau_{v,i,t}$ and reducing the noise variance and thus its uncertainty about z_t.

Timeline of Events. At the beginning of each period, aggregate productivity Z_t is realized. However, agents cannot observe the actual value of Z_t. The problem facing the intermediate goods firm consists of three stages:

1. *Information Acquisition Choice.* Each firm i chooses the precision of its signal .
2. *Pricing Choice.* Based on the precision of the signal chosen in the first stage, each firm receives a signal $s^z_{i,t}$. Based on the signal $s^z_{i,t}$, firms form beliefs about z_t based on their information set at time t, $\mathcal{I}_{i,t} = \{s^z_{i,t}\}$. Each firm chooses its price based on its beliefs.
3. *Production Choice.* The true z_t is revealed, and firms choose labor $N_{i,t}$.

Markets clear at the end of each period. I solve the firm's problem by backward induction. In stage 3, each firm minimizes its cost by choosing labor. In stage 2, each firm's pricing choice occurs under imperfect information. Each firm's information set in period t is given by $\mathcal{I}_{i,t} = \{s^z_{i,t}\}$, which depends on the signal received. The firm's profit-maximization problem is given by:

$$\max_{P_{i,t}} \mathbb{E}_{i,t}\left[\left(P_{i,t} - \frac{W_t}{Z_t}\right) P_{i,t}^{-\epsilon} Y_t | \mathcal{I}_{i,t}\right] \tag{14}$$

The optimal price of firm i is given by:

$$P_{i,t} = \frac{\epsilon}{\epsilon - 1} W_t \mathbb{E}_{i,t}\left(\frac{1}{Z_t}\right) \tag{15}$$

The optimal price equals a constant monopolistic markup over expected real marginal costs. As shown in Appendix A, after summing both sides of Eq. (15), I obtain the following expression for aggregate output:

$$Y_t = \left(\int_0^1 \mathbb{E}_{i,t}\Big(\frac{1}{Z_t}\Big)^{1-\epsilon} \mathrm{di}\right)^{\frac{1}{\epsilon-1}} N_t = \exp\Big(\hat{z} - \frac{1}{2\tau_z} + \frac{1}{2\tau_z}\epsilon \frac{1}{1 + \frac{\tau_z}{\tau_{v,t}}}\Big) N_t \tag{16}$$

where $\hat{z} = \mathbb{E}_t(z_t|\mathcal{I}_t)$ is the aggregate expectation of z_t conditional on the aggregate information set.[11] Eq. (16) shows that output is decreasing in uncertainty in the

[11] Aggregate expectations are averaged over the integral of each firm's expectations. More details and derivations are provided in the Appendix.

sense that it is increasing in the precision of the signal $\tau_{v,t}$.[12] When firms make their pricing decisions under uncertainty, they deviate from first-best optimal pricing under full information. As uncertainty increases, these deviations from this approach increase and output falls. Thus, shocks to $\tau_{v,t}$ generate counter-cyclical movements in uncertainty.

In stage 1, firms acquire information to reduce the level of uncertainty in the signal. In order to maximize expected profits in stage 1, it is useful to express firm i's realized profits in stage 3, as follows:

$$\Pi_{i,t} = \left(P_{i,t} - \frac{1}{Z_t}W_t\right)Y_{i,t} = \frac{1}{\epsilon}Y_t\frac{\mathbb{E}_{i,t}(\frac{1}{Z_t})^{1-\epsilon}}{\int_0^1 \mathbb{E}_{i,t}(\frac{1}{Z_t})^{1-\epsilon}\mathrm{d}i} \tag{17}$$

After taking expectations of Eq. (17) over all possible signal realizations, firm i's expected profits are given by:

$$\Pi^E_{i,t} = \frac{1}{\epsilon}Y_t\frac{\exp\left\{(\epsilon-1)(\hat{z} - \frac{1}{2\tau_z} + \frac{1}{2\tau_z}\epsilon\frac{1}{1+\frac{\tau_z}{\tau_{v,i,t}}})\right\}}{\exp\left\{(\epsilon-1)(\hat{z} - \frac{1}{2\tau_z} + \frac{1}{2\tau_z}\epsilon\frac{1}{1+\frac{\tau_z}{\tau_{v,-i,t}}})\right\}} \tag{18}$$

Eq. (18) shows that a firm's expected profits are increasing in Y_t and decreasing in its uncertainty. When the economy is booming, expected profits are higher. In addition, as the precision of the signal improves and uncertainty falls, this reduces the occurrence of mispricing. Hence, this also leads to an increase in expected profits. Since expected profits depend only on the aggregate state and prior beliefs, which are common across all firms, I now drop i subscripts for each firm's information acquisition choice.[13] The formal definition of equilibrium in the model is as follows.

Definition 1. *A Rational Expectations Equilibrium (REE) is a sequence of aggregate allocations* $\{C_t, N_t, Y_t, \Pi_t, \tau_{v,t}\}$*, individual productions* $Y_{i,t}$ *for intermediate firms, and prices,* $\{W_t.\{P_{i,t}\}\}$*, such that for each realization of* A_t*:*

1. *C_t and N_t maximize household's utility, given the equilibrium wage W_t.*
2. *Equation (10) maximizes the final goods firm's profit, given equilibrium prices $\{P_{i,t}\}$.*
3. *Given W_t and signals $s^z_{i,t}$, $P_{i,t}$ maximizes the expected profits for an intermediate firm.*

[12] Since all firms face the same aggregate state, I show that all firms will make the same information acquisition choice. Hence, the precision of the chosen signal is the same for all firms, and aggregate output depends on the precision of the common signal across all firms. As a result, I drop the i subscript when computing aggregate output.

[13] A source of heterogeneity that each firm faces is the idiosyncratic noise $v_{i,t}$ of the signal. Therefore, pricing and production choices remain heterogeneous as they depend on the signals received.

4. *$\tau_{v,i,t}$ solves the intermediate firm's information acquisition problem by maximizing expected profits.*

5. *All markets clear, namely, $Y_t = C_t$ and $N_t = \int_0^1 N_{i,t} di$.*

3.2 The Information Acquisition Problem

I now describe the firm's problem in stage 1 of the model. A firm increases its expected profits by acquiring more information. I define information as outlined in Shannon (1948):

$$I(z_t; s_{i,t}^z) = \log_2 \left(\frac{Var(z_t)}{Var(z_t | s_{i,t}^z)} \right) = \log_2 \left(\frac{\tau_{v,i,t}}{\tau_z} + 1 \right) \tag{19}$$

where $I(z_t; s_{i,t}^z)$ represents mutual information. Mutual information is a measure of uncertainty reduction. Firms acquire more information by choosing a higher value of $\tau_{v,i,t}$, which results in a lower value of $Var(z_t|s_{i,t}^z)$. This implies a larger amount of mutual information and a greater reduction in uncertainty. When the signal is not informative at all and $\tau_{v,i,t}$ equals 0, $Var(z_t|s_{i,t}^z)$ equals the unconditional variance of z_t, $Var(z_t)$. In this case, mutual information equals zero, and there is no reduction in uncertainty.

The frictions in the information acquisition model consist of two major components: a rational inattention component and an information search frictions component. In the rational inattention component, each firm faces information processing costs of $\theta_I I(z_t; s_{i,t}^z)$, where θ_I is the unit cost of processing information. This is similar to rational inattention models such as Woodford (2009).

The novelty of this paper is to introduce information search frictions. In addition to information processing costs, each firm incurs information search costs with a unit cost of θ_S. Search costs represent the idea that firms must exert effort to search for or acquire information they require. Each firm's maximization problem is thus given by:

$$\max_{I_t, S_t} \Pi_t^E - \underbrace{\theta_I I(z_t; s_{i,t}^z)}_{\substack{\text{Rational Inattention} \\ \text{(Information Processing Costs)}}} + \underbrace{\theta_S S_t}_{\substack{\text{Information Search Frictions} \\ \text{(Information Search Costs)}}} \tag{20}$$

subject to

$$\underbrace{\underbrace{I(z_t; s^z_{i,t})}_{\substack{\text{Yield of search intensity} \\ \text{in terms of entropy reduction} \\ \text{(or Mutual Information)}}} = \underbrace{D_t(z_t)}_{\substack{\text{Supply of Information} \\ \text{or ``data'' (Farboodi} \\ \text{and Veldkamp, 2021)}}} \cdot \underbrace{S_t^{\alpha_S}}_{\text{Demand for Information}}}_{\text{Information Search Frictions}} \tag{21}$$

Eq. (21) shows that firms obtain more mutual information by increasing search intensity S_t. I denote the behavior of searching for information as the "demand for information". I assume α_S to be less than 1, which implies decreasing returns to search intensity. Consider an individual who searches for information about "coronavirus" on the internet. This generates numerous articles containing information. Since the individual has no prior knowledge about the term "coronavirus", he obtains information (measured in mutual information) about the subject. However, as he moves on to the next article, this may repeat information from the previous article. Hence, the marginal gain of mutual information from reading an additional article decreases. This intuitively illustrates the idea that the yield in terms of mutual information can exhibit diminishing returns in search intensity.

The yield of search intensity also depends on the "supply of information". Farboodi and Veldkamp (2019) define information supply as data abundance in the economy. According to Farboodi and Veldkamp (2019), information supply is modeled as a by-product of output. The idea is that as economic activity increases, this leads to a larger sample of data points in the economy, increasing the amount of information. Since output depends on the state of the economy, I assume that information supply depends on z_t, such that $D_t = z_t^{\alpha_D}$.[14]

3.3 Rational Inattention Model

I start by considering a version of the model with rational inattention in the absence of information search frictions. I show how this model is inconsistent with the evidence suggesting both counter-cyclical information acquisition and uncertainty. In this case, θ_S is equal to zero and equation (21) does not hold. In this information acquisition problem, firms choose the amount of mutual information $I(z_t; s^z_{i,t})$ to maximize expected profits net of information processing costs.

Definition 2. *In the rational inattention model, information acquisition is defined as mutual information: $I(z_t; s^z_{i,t})$.*

In the rational inattention model, I define information acquisition as the quantity

[14] This assumption streamlines the analytical solution without losing any intuition. I allow information supply to depend on Y_t in the quantitative section.

of mutual information, $I(z_t; s^z_{i,t})$. Higher uncertainty (or a lower reduction in uncertainty) implies that less information is acquired. Hence, in the rational inattention model, there is a direct inverse relationship between information acquisition and the level of uncertainty.

Pro-cyclical Information Acquisition. I now show that the rational inattention problem generates counter-cyclical uncertainty and pro-cyclical information acquisition. The first order condition of the firm's maximization problem in stage 1 is given by:

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s^z_{i,t})} = \theta_I \tag{22}$$

Eq. (22) equates the marginal benefit to the marginal cost of acquiring information. As shown in Figure 4, the marginal benefit of acquiring information is decreasing in I_t because expected profits are concave in I_t. This is due to the convex costs of posting a sub-optimal price that differs from the first-best price under full information. The marginal cost of acquiring information equals the unit cost of processing information. The initial equilibrium of mutual information, or information acquisition, is given by the interaction of the marginal cost and marginal benefit curves at E_0. After log-linearizing Eq. (16) and Eq. (22), output and mutual information are jointly determined by the following equations:

$$\underbrace{\tilde{y}_t - \phi_1 \tilde{I}_t}_{\substack{\text{Marginal Benefit of} \\ \text{Acquiring Information}}} = 0 \quad \text{(Information Acquisition)} \tag{23}$$

$$\tilde{y}_t = \eta_1 \tilde{I}_t + \eta_2 \tilde{z}_t \quad \text{(Production)} \tag{24}$$

where Eq. (23) and Eq. (24) are the log-linearized equations of Eq. (22) and Eq. (16) respectively; $\tilde{x}_t$ denotes a variable's log deviation from its steady state, and ϕ_1, η_1, and η_2 are strictly positive parameters. Before solving for optimal output and mutual information, it is useful to specify the parameter restrictions that lead to a stable equilibrium. All proofs are relegated to the appendix.

Lemma 1. *Assume that $|\phi_1| > \eta_1 > 0$. Then, a unique stable log-linear equilibrium exists in the rational inattention model.*

I now solve for equilibrium output and mutual information in Proposition 1.

Proposition 1. *In equilibrium, mutual information and output are functions of z_t, as follows:*

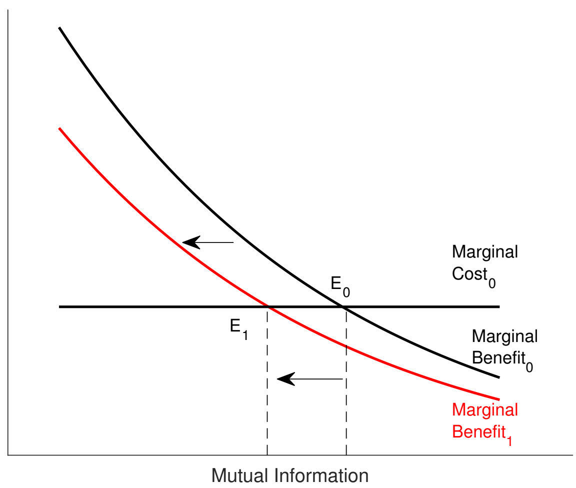
$$\tilde{I}_t = \frac{1}{\phi_1 - \eta_1} \eta_2 \tilde{z}_t \tag{25}$$

$$\tilde{y}_t = \frac{\phi_1}{\phi_1 - \eta_1}\eta_2 \tilde{z}_t \tag{26}$$

where ϕ_1, η_1, *and* $\eta_2 > 0$.

Proposition 1 states that information acquisition is pro-cyclical, which generates counter-cyclical uncertainty due to the inverse relationship between uncertainty and information acquisition. Consider a negative shock to z_t. This decreases the expected profits and output of each firm. As expected profits fall, each firm's expected marginal product of inputs declines, decreasing the marginal benefit of acquiring information. This leads to a leftward shift of the marginal benefit curve in Figure 4. As such, the equilibrium mutual information decreases from E_0 to E_1.

Figure 4: Pro-cyclical Information Acquisition and Counter-cyclical Uncertainty



Notes: This figure plots the marginal benefit and marginal cost of acquiring $I(z_t; s^z_{i,t})$ and shows their responses to a negative shock z_t.

However, the empirical evidence presented by Flynn and Sastry (2021) shows that information acquisition behavior is counter-cyclical. Moreover, in the rational inattention context, information acquisition is defined as mutual information, so counter-cyclical information acquisition would imply procyclical uncertainty.[15] Next, I examine how the cyclicality of mutual information affects the amplification of productivity shocks under rational inattention.

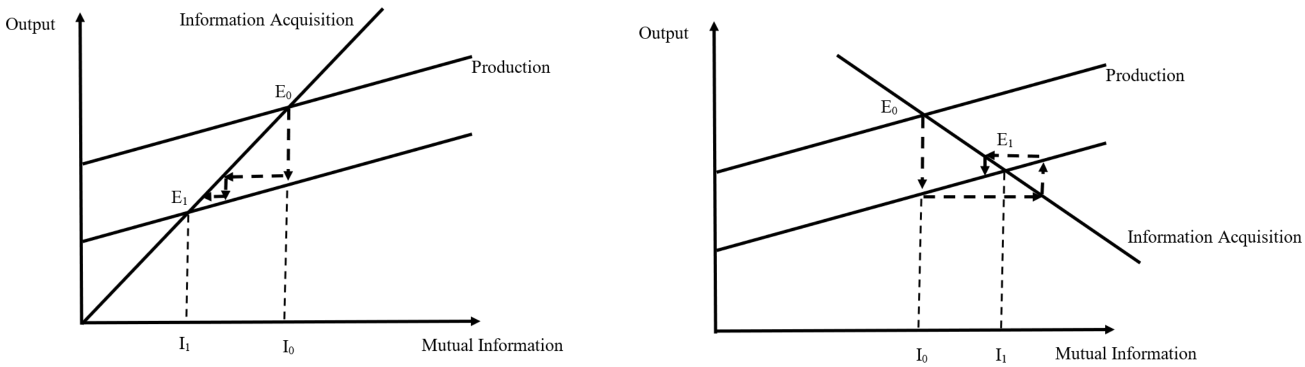
[15]Counter-cyclical information acquisition and uncertainty cannot co-exist in a framework where uncertainty is endogenously determined. However, they can co-exist under exogenous fluctuations of uncertainty. Under this environment, counter-cyclical information acquisition is a response to positive uncertainty shocks, which lowers uncertainty. As a result, this leads to a dampening of recessions. Hence, a model with counter-cyclical information acquisition (or mutual information) cannot quantitatively match uncertainty dynamics.

Corollary 1. *Define the uncertainty multiplier as* $\frac{\partial \tilde{y}_t}{\partial \eta_2 \tilde{z}_t} = \frac{\phi_1}{\phi_1 - \eta_1}$. *If mutual information is pro-cyclical* ($\phi_1 > 0$), *then the uncertainty multiplier is greater than 1. If mutual information is counter-cyclical* ($\phi_1 < 0$), *then the uncertainty multiplier is less than 1.*

Corollary 1 shows that the cyclicality of mutual information affects the amplification of output. This is evident from the uncertainty multiplier, defined as the elasticity of output to a shock originating to $\eta_2 \tilde{z}_t$.[16] When the uncertainty multiplier is greater than one, output responds more than one-for-one to a productivity shock.

Figure 5 plots the information acquisition and production equation given by Eq. (23) and (24) respectively. A negative shock to $\tilde{z}_t$ shifts the production line downwards. The left panel shows the case of procyclical information. In this case, lower production decreases profits, which decreases the incentive to acquire information. Lesser mutual information increases uncertainty, which further depresses output. This leads to an amplification loop between output and mutual information.

Figure 5: Interaction between Uncertainty Multiplier and Cyclicality of Information Acquisition



(a) Uncertainty Multiplier > 1 (b) Uncertainty Multiplier < 1

Notes: This figure presents the impact of a negative shock to $z_t$on the uncertainty multiplier dynamics. The left-hand panel depicts the model with pro-cyclical information acquisition, while the right-hand panel depicts the model with counter-cyclical information acquisition.

By contrast, when mutual information is counter-cyclical, the uncertainty multiplier is less than one. This implies that the output response is dampened compared to its initial shock originating from $\eta_2 \tilde{z}_t$. As shown in the right panel of Figure 5, counter-cyclical mutual information implies a rise in mutual information in response to an adverse shock. This counteracts the effect of the initial shock and dampens recessions.

Signal Structure (Rational Inattention Model). Next, I show how the rational inattention model relates to the noisy information model presented in Coibion and Gorod-

[16] η_2 is equivalent to the speed of learning when using the Kalman filter. Hence, $\eta_2 \tilde{z}_t$ is equivalent to $\hat{z}_t$.

nichenko (2015). In the rational inattention model, economic agents internalize that each signal consists of two components: the realization and noise components. The realization component consists of the actual value of z_t, while the noise component consists of noise resulting from uncertainty in the rational inattention model. Hence, the signal $s^z_{i,t}$ takes the following form:

$$s^z_{i,t} = \underbrace{z_t}_{\substack{\text{Realization} \\ \text{Component}}} + \underbrace{v_{i,t}}_{\substack{\text{Noise} \\ \text{Component}}} \tag{27}$$

Equation (27) is identical to the signal structure presented in Coibion and Gorodnichenko (2015). When firms acquire information in the rational inattention model, this corresponds to a reduction in the variance of $v_{i,t}$. In addition, information quality is perfect in the rational inattention model, which implies that the realization component of the signal equals the actual hidden state z_t, so that the signal is an unbiased estimate of the true hidden state. The only source of fluctuations in uncertainty that agents in the rational inattention model face originate from fluctuations in the variance of $v_{i,t}$. Because information acquired I^*_t does not eliminate all uncertainty about z_t, the noise component captures the residual uncertainty in the rational inattention model.

The rational inattention model predicts pro-cyclical information acquisition, which implies an increase in the variance of the noise component during downturns. However, Coibion and Gorodnichenko (2015) document lower information rigidities in recessions, which implies a lower variance of $v_{i,t}$. Hence, the rational inattention model cannot replicate observed empirical facts on the cyclicality of information acquisition and uncertainty.

3.4 Rational Inattention + Information Search Frictions

To reconcile the co-existence of counter-cyclical information acquisition and uncertainty, I introduce information search frictions in the firm's information acquisition problem. In the rational inattention model, firms maximize their payoffs by choosing mutual information. By contrast, in the model with search frictions, firms maximize their payoffs by searching for information and choosing information search intensity S_t. Their expected profits determine their total benefit of searching for information, Π^E_t, which is identical to that in the rational inattention model. However, in this case, when searching for information, they incur both information processing costs θ_I and information search costs θ_S.

In addition to information search costs, they face the constraint given by Eq. (21), which connects mutual information to information supply and information search intensity.

Counter-cyclical Information Acquisition and Uncertainty. Next, I demonstrate how the model with search frictions generates counter-cyclical information acquisition.

Definition 3. *In the model with search frictions, information acquisition is defined as information search intensity:* S_t*.*

In the model with search frictions, I define information acquisition behavior as information search intensity rather than the quantity of mutual information obtained. Under this definition, an increase in information acquisition or information search intensity (S_t) does not necessarily mean a decline in uncertainty. In other words, unlike the rational inattention model, there is now a disconnect between information acquisition or information search intensity (S_t) and mutual information (I_t), where mutual information is a direct measure of uncertainty reduction. This is because mutual information does not solely depend on search intensity but also on the information supply. The first order condition governing the choice of S_t is:

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} = \theta_I + \theta_S z_t^{-\alpha_D} S_t^{1-\alpha_S} \tag{28}$$

The left and right-hand sides of Eq. (28) denote the marginal benefit and marginal cost of information search intensity, respectively. After log-linearizing Eq. (16), Eq. (21), and Eq. (28), output, mutual information, and information search intensity are jointly determined by the following equations:

$$\tilde{y}_t - \phi_1 \tilde{I}_t = -\phi_2 \tilde{z}_t + \phi_3 \tilde{s}_t \quad \text{(Information Acquisition)} \tag{29}$$

$$\tilde{y}_t = \eta_1 \tilde{I}_t + \eta_2 \tilde{z}_t \quad \text{(Output Production)} \tag{30}$$

$$\tilde{I}_t = \psi_1 \tilde{z}_t + \psi_2 \tilde{s}_t \quad \text{(Information Production)} \tag{31}$$

Consider a fall in z_t. This leads to two different effects on Eq. (29). The first effect on the marginal benefit of search intensity is identical to its counterpart in the rational inattention model. A fall in z_t reduces expected profits which causes a decrease in the expected marginal product of inputs and a decline in the marginal benefit of information acquisition or information search intensity. I denote this as the expected profit effect.

A fall in z_t also affects information supply, affecting the level of mutual information in the economy. I denote this as the information quality effect. Information quality affects Eq. (29) through ϕ_1 and ϕ_2. The direction of this effect depends on the net value of acquiring information: on the one hand, the information quality effect leads to a rise in the marginal benefit of acquiring mutual information ($\phi_1 \tilde{I}$) as mutual information decreases due to a decline in information supply; on the other hand, it leads to a rise in the marginal costs of acquiring information ($\phi_2 \tilde{z}_t$), as each firm needs to search more to obtain a given unit of mutual information.

The overall effect of information quality on information search intensity depends on the change in the net value of acquiring information. If the rise in the marginal benefit exceeds the increase in the marginal costs of information acquisition, then the information quality effect leads to an increase in the net value of acquiring information when z_t falls. This tends to generate counter-cyclical search intensity. Conversely, if the net value of information acquisition decreases when z_t falls, then search intensity will tend to be procyclical. Before solving for optimal mutual information and search intensity, it is useful to define the parameter restrictions that lead to a stable equilibrium.

Lemma 2. *Assume that $\phi_1\psi_2 + \phi_3 > \phi_1\psi_2\eta_1$. Then, a unique stable log-linear equilibrium exists in the model with search frictions.*

I now solve for equilibrium search intensity and mutual information.

Proposition 2. *In equilibrium, mutual information and search intensity depend on z_t as follows:*

$$\tilde{s}_t = B_t^s \tilde{z}_t = \frac{\eta_1\psi_1 + \eta_2 - \phi_1\psi_1 + \phi_2}{\phi_3 + \phi_1\psi_2 - \eta_1\psi_2}\tilde{z}_t \tag{32}$$

$$\tilde{I}_t = B_t^I \tilde{z}_t = \frac{\eta_2\psi_2 + \phi_2\psi_2 + \psi_1}{\phi_1\psi_2 + \phi_3 - \phi_1\psi_2\eta_1}\tilde{z}_t \tag{33}$$

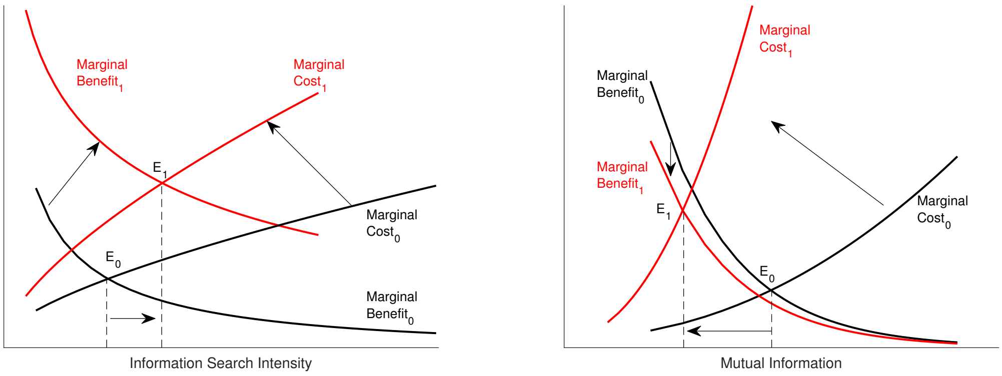
where ϕ_1, ϕ_2, ϕ_3, η_1, η_2, ψ_1, and $\psi_2 > 0$.

Proposition 2 highlights the conditions necessary to generate counter-cyclical search intensity. As previously discussed, the numerator captures the expected profit channel as $\eta_1\psi_1 + \eta_2$. The net value of acquiring information due to the information quality channel is captured as the difference between the marginal benefit $\phi_1\psi_1$ and the marginal cost ϕ_2.

Corollary 2. *Mutual information is pro-cyclical, that is, $\frac{\partial \tilde{I}_t}{\partial \tilde{z}_t} > 0$. If $\phi_1 > \frac{\eta_1\phi_2 + \eta_2 + \phi_2}{\psi_1}$, then information search intensity is counter-cyclical, that is, $\frac{\partial \tilde{s}_t}{\partial \tilde{z}_t} < 0$.*

Corollary 2 states that if the marginal benefit of acquiring information rises by a large amount when z_t falls due to scarcity of data operating through ϕ_1, then search intensity $\tilde{s}_t$ will be counter-cyclical. By contrast, mutual information is unambiguously pro-cyclical in this model.

Figure 6: Effect of a Negative Shock to z_t on Search Intensity and Mutual Information



(a) Counter-cyclical Search Intensity (b) Counter-cyclical Uncertainty

Notes: This figure presents the impact of a negative shock to z_t to information search intensity (left panel) and mutual information (right panel).

Figure 6 (a) plots a scenario in which information search intensity rises in response to an adverse total factor productivity (TFP) shock. When the information quality effect (ϕ_1) dominates the expected profit effect, the marginal benefit of search intensity shifts to the right. At the same time, the marginal cost curve shifts upward (ϕ_2) due to the decline in information supply when z_t falls. In this case, when there is a positive net value of acquiring information due to the information quality channel, information search intensity rises from E_0 to E_1 in response to a fall in z_t.

Figure 6 (b) plots the response of mutual information to a shock to z_t. The expected profit effect shifts the marginal benefit curve downward.[17] In addition, the marginal cost curve shifts upward. This implies that mutual information falls unambiguously, which implies counter-cyclical uncertainty.

Information Quality. Denote $I(D_t, S_t)$ as mutual information acquired in the model with search frictions and $I(\bar{D}, S_t)$ as mutual information when the supply of information is held fixed. Then $I(D_t, S_t)$ can be decomposed as

$$I(D_t, S_t) = \underbrace{I(\bar{D}, S_t)}_{\substack{\text{Mutual Information} \\ \text{Unadjusted for Quality}}} + \underbrace{I(D_t, S_t) - I(\bar{D}, S_t)}_{\text{Adjusting for Quality}} \tag{34}$$

[17]Since mutual information is plotted on the horizontal axis, the fall in mutual information due to the information quality effect does not shift the marginal benefit curve. This effect leads to a *movement* along the marginal benefit curve.

When firms increase mutual information by increasing their search intensity while holding the supply of information constant, this generates a payoff of $I(\bar{D}, S_t)$. However, after accounting for fluctuations in information supply, mutual information falls from $I(\bar{D}, S_t)$ to $I(D_t, S_t)$. In this case, I define $I(\bar{D}, S_t)$ as mutual information unadjusted for information quality and $I(D_t, S_t)$ as mutual information adjusted for quality. Imperfect and counter-cyclical information quality implies that $I(D_t, S_t) - I(\bar{D}, S_t)$ is negative during recessions, as low quality information can contain inaccuracies that lead to an increase in uncertainty (or a decrease in mutual information).

Definition 4. *Information quality is defined as χ_t^{model}, which satisfies*

$$\hat{I}_t = I(D_t, S_t) = \chi_t^{model} I(\bar{D}, S_t) \tag{35}$$

where χ_t^{model} is restricted to be less than or equal to one, and $\bar{D}$ satisfies $I(\bar{D}, S_t) = I_{ss}^*$, where I_{ss}^* is the steady state mutual information in the rational inattention model without search frictions.

I define the wedge χ_t^{model} as a measure of information quality. When χ_t^{model} equals one, then information quality is perfect in the sense that the information acquired does not contain any inaccuracies. As such, $I(D_t, S_t) - I(\bar{D}, S_t)$ equals zero, and there is no adjustment for information quality. Hence, perfect information quality means lower uncertainty. In contrast, when χ_t^{model} is less than one, then information quality is imperfect, which results in a rise in uncertainty or a decrease in mutual information from $I(\bar{D}, S_t)$ to $I(D_t, S_t)$.

Signal Structure (Rational Inattention + Search Frictions). Next, I show how information quality can be incorporated into the signal structure framework.

Proposition 3. *Suppose that $I(\bar{D}, S_t)$ satisfies*

$$s_{i,t}^z = z_t + v_{i,t} \tag{36}$$

Then there exists a random variable χ_t^{emp}, such that $\hat{I}_t$ satisfies

$$s_{i,t}^z = \underbrace{\chi_t^{emp} z_t}_{\substack{\textit{Realization} \\ \textit{Component}}} + \underbrace{v_{i,t}}_{\substack{\textit{Noise} \\ \textit{Component}}} \tag{37}$$

where χ_t^{emp} has finite variance $\sigma_t^{\chi,emp}$ and mean equal to one.

Proposition 3 states that mutual information in the model with search frictions satisfies a signal structure given by Eq. (37). In this signal structure, there is an additional random variable χ_t^{emp} in the realization component, which captures the notion of imperfect information quality. As a result, residual uncertainty generated by observing the signal in Eq. (37) is strictly higher than that generated by the signal in Eq. (36). This is due to the fact that $\hat{I}$ is less than $I(\bar{D}, S_t)$ and χ_t^{model} is restricted to be less than one. In this scenario, even if a forecaster receives a signal with $v_{i,t}$ equal to zero, he does not necessarily receive an accurate signal due to χ_t^{emp}, which fluctuates away from one due to imperfect information quality. As a result, residual uncertainty ($Var(z_t|s_{i,t})$) remains even if σ_t^v is zero.

Proposition 4. *Assume that χ_t^{emp} is a random variable with $\sigma_t^{\chi,emp}$ and mean equal to one. Then $\sigma_t^{\chi,emp}$ is decreasing in χ_t^{model}.*

Proposition 4 states that there is a direct mapping between χ_t^{emp} and χ_t^{model}. The variance of χ_t^{emp} increases as χ_t^{model} decreases. This is because $\hat{I}$ falls as χ_t^{model} falls while $I(\bar{D}, S_t)$ remains fixed. As such, the probability of χ_t^{emp} deviating further from one increases as the model-based measure of information quality χ_t^{model} decreases. This implies that the median forecaster receives an increasingly inaccurate signal when information quality is lower, even though he receives a signal with zero noise ($v_{i,t} = 0$). Hence, χ_t^{emp} is a relevant measure of information quality. This is also consistent with the empirical measure of information quality discussed in the previous section.

Lower Information Quality in Downturns. Next, I examine changes in information quality over the business cycle.

Proposition 5. *If S_t is counter-cyclical, then χ_t^{model} is strictly increasing in z_t.*

Proposition 5 states that information quality is lower in downturns. A decrease in z_t thus leads to an unambiguous decrease in mutual information $\hat{I}$. In contrast, $I(\bar{D}, S_t)$ considers mutual information when the information supply (or data) is held fixed. Counter-cyclical information search intensity implies that $I(\bar{D}, S_t)$ increases in downturns. As a result, the gap between $I(\bar{D}, S_t)$ and $\hat{I}$ widens during a downturn, leading to a fall in χ_t^{model}.

Intuitively, as firms search more for information, this increases mutual information $I(\bar{D}, S_t)$ when not accounting for quality. However, because information quality is lower, such information contains more inaccuracies in a downturn. Overall, this leads to a decline in $\hat{I}$ and a rise in uncertainty after accounting for information quality.

From the signal structure in Eq. (37), the decline in information quality leads to an increase in the variance of χ_t^{emp}. The net effect is an increase in $\text{Var}(z_t|s_{i,t}^z)$. Hence, the signal structure of the model with search frictions can rationalize the existence of counter-cyclical information acquisition and uncertainty, unlike models with rational inattention.

3.5 Comparison Between Models

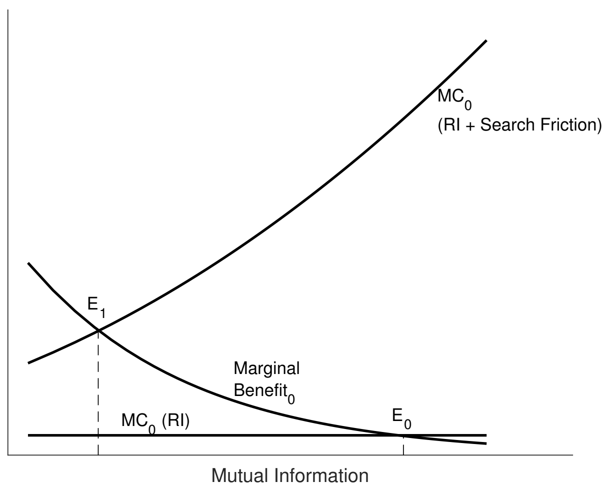
I now compare the rational inattention model and the model with information search frictions and demonstrate why it is necessary to introduce information quality, even though one could obtain similar amplified dynamics of output and uncertainty with pro-cyclical information acquisition in the rational inattention model.

Proposition 6. *When $\theta_S > 0$, Mutual information in the model with both rational inattention and search frictions is strictly less than its counterpart in the rational inattention model, that is,*

$$\hat{I}_t < I_t^* \tag{38}$$

where $\hat{I}_t$ and I_t^* are mutual information obtained in the model with search frictions and the rational inattention model, respectively.

Figure 7: Information Quality and the Importance of θ_s



Notes: This figure plots the marginal benefit and marginal cost of acquiring $I(z_t; s^z_{i,t})$ for both the rational inattention model and the model with search frictions.

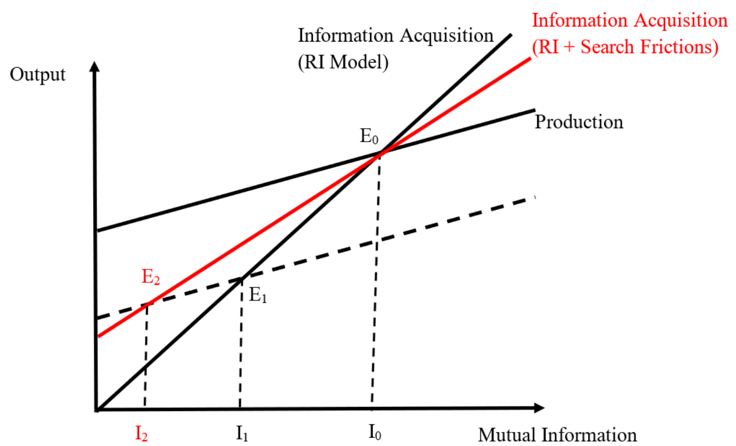
Denote I^* and $\hat{I}$ as mutual information in the rational inattention model and the model with search frictions, respectively. Proposition 6 shows that whenever information search frictions are present, and θ_S is greater than zero, the level of mutual information in the model with search frictions, $\hat{I}_t$ is less than the level of mutual information in the rational inattention model I_t^*. This implies that the level of mutual information in the model with search frictions is inefficient compared to the rational inattention model. Figure 7 illustrates this fact as the marginal cost of search intensity in the model with search frictions is strictly higher than that of the rational inattention model, which implies that $\hat{I}_t$ (E_1) is strictly lower than I_t^* (E_1).

Lower Information Quality Due to Search Frictions. Next, I examine how information quality is affected by the presence of search frictions by comparing it with the rational inattention model. The model-based measure of information quality can be decomposed in the following way.

$$\chi_t^{\text{model}} = \frac{\hat{I}}{I(\bar{D}, S_t)} = \underbrace{\frac{\hat{I}}{I^*}}_{\substack{\text{Information Quality} \\ \text{Due to Search Frictions}}} \frac{I^*}{I(\bar{D}, S_t)} \tag{39}$$

Consider a fall in z_t, which leads to a fall in expected profits in the rational inattention model. However, as discussed in the previous section, a rise in marginal cost and counter-cyclical search intensity lead to a larger fall in $\hat{I}$ compared to I^*. As a result, $\frac{\hat{I}}{I^*}$ necessarily falls in a downturn. Hence, the decline in information quality χ_t^{model} can be attributed to an increase in inefficiency resulting from the difference in mutual information between the rational inattention model and the model with search frictions.

Figure 8: Information Quality (RI vs RI + Search Frictions)



Notes: This figure plots the production and information acquisition equation. The red line denotes the information acquisition equation in the model with search frictions. The production equation shifts downwards due to a negative TFP shock.

This is evident in Figure 8. Search frictions add additional fluctuations in mutual information due to changes in information supply. Hence, the information acquisition line is flatter for the rational inattention model with search frictions than the rational inattention model. Denote the initial equilibrium as E_0. A fall in z_t shifts the production line downwards. In the rational inattention model, the new equilibrium is at E_1. By contrast, in the rational inattention model with search frictions, the new equilibrium is at E_2. From the figure, E_2 generates lower output and mutual information than E_1. Therefore, even though one can obtain similar amplified dynamics of output and uncertainty with pro-cyclical information acquisition in the rational inattention model, the model with search frictions generates larger fluctuations in uncertainty and deeper recessions due to fluctuations in information supply and quality.

4 Model Calibration and Estimation

Next, I embed information quality in a quantitative New Keynesian model. This section discusses the calibration and estimation of the model.

4.1 Additional Ingredients

I extend the model of Section 3 to a New Keynesian model in the spirit of Christiano et al. (2005) and Smets and Wouters (2007). Firms face capital adjustment costs and Calvo price stickiness. The estimated model also features monetary policy and shocks to TFP, investment efficiency, price markups, preferences, monetary policy, and mutual information (or uncertainty). More details are provided in the Appendix.

4.2 Calibration and Estimation Strategy

I split the parameters into two categories, Ξ_1 and Ξ_2. The parameters in Ξ_1 are calibrated externally. The parameters in Ξ_2 are then estimated using Bayesian methods. Ξ_1 consists of the following parameters

$$\Xi_1 : \{\beta, \eta, \alpha, \delta, \epsilon\}$$

The discount rate β is set to 0.99. I assume an infinitely elastic labor supply ($\eta = 0$). The capital income share α is set to 0.33. The capital depreciation rate δ is set to 0.025 at a quarterly frequency. I set the elasticity of substitution ϵ to be 4, which implies an average markup of $\frac{4}{3}$. Table 1 shows these parameters.

Next, I estimate the remaining parameters using Bayesian methods, using the following set of seven observables from 2004Q1 to 2020Q2.[18,19]

$$\{\Delta \log Y_t, \Delta \log C_t, \Delta \log I_, \log \pi_t, \log(1+i_t), \Delta \log S_t, \Delta \log Var(z_t|s^z_{i,t})\}$$

where $Y_t, C_t, I_t, \pi_t, 1+i_t, S_t, andVar(z_t|s^z_{i,t})$ are 1) output, 2) consumption, 3) investment, 4) the inflation rate, 5) the nominal interest rate, 6) information search intensity, and 7) uncertainty, respectively.[20]

Table 1: Parameters from External Calibration

Parameters Set Independently			
Interpretation	Symbol	Value	Source
Household Discount Rate	β	0.99	Standard Literature
Labor Supply Elasticity	η	0	
Capital Income Share	α	0.33	
Depreciation	δ	0.025	
Elasticity of Substitution	ϵ	4	

Notes: This table shows the parameters that are set externally in the model with rational inattention and information search frictions.

I measure output as real GDP and consumption as real personal consumption expenditure on non-durable goods and services. I measure investment as the sum of private fixed investment on all types of fixed assets and personal consumption expenditure on durable goods. The inflation rate is the quarterly percentage change of the GDP deflator. I use the effective Federal funds rate for the interest rate. Information search intensity is measured by the average Google search shares of the top 20 major US media companies in the Business and Industrial category. For uncertainty, I rely on the measure in Jurado et al. (2015) as their definition of uncertainty is identical to my model counterpart.

Discussion of Mutual Information Shocks. As the model features constant TFP volatility, shocks to mutual information map into uncertainty shocks, where uncertainty is defined as the perceived variance of TFP, conditional on the signal received. This is consistent with the empirical measure of uncertainty.

In a robustness exercise, I consider another measure of uncertainty, defined as volatility shocks to the true hidden state. I show that volatility shocks to the hidden

[18]For a detailed description of the observables, see the appendix.

[19]The estimation period begins in 2004Q1 due to the availability of the information search intensity measure.

[20]Since there are 7 observables with 6 shocks, I introduce measurement error in uncertainty.

state are quantitatively equivalent to mutual information shocks. I also demonstrate that even though the quantity of mutual information may differ from the model discussed in this paper, output and uncertainty allocations remain identical even in the case of volatility shocks to the hidden state. Hence, volatility shocks are isomorphic to mutual information shocks.

Production of Mutual Information. I assume that the returns to data take the following form:

$$D_t = Y_t^{\alpha_D} \tag{40}$$

I assume decreasing returns to scale for the production of mutual information and set the priors of α_S (returns to search intensity) and α_D (returns to data) to be less than one.[21] The priors for information processing and search costs are equal to 0.003, which would imply pro-cyclical search intensity. Given the empirical measure of information search intensity, the posterior mode generated for information processing and search costs imply counter-cyclical search intensity instead. This is because the Bayesian estimation maps the model to the observables. If the model predicted pro-cyclical information search intensity while Google search shares are counter-cyclical, this would generate a low likelihood function value, as the model would require shocks at the extreme ends of the normal distribution. Hence, to maximize the likelihood function value, the estimated parameters will lie in the space that generates counter-cyclical search intensity.

The prior means and standard deviations for the rest of the shock processes are set to the conventional values in Smets and Wouters (2007). To conduct Bayesian estimation, I obtain 1,000,000 draws, discard the first 25% as burn-in, and use the remaining draws as posteriors.

Table 2 shows the estimated parameters using Bayesian methods. As expected, with α_S less than one, the yield of information exhibits diminishing returns to search intensity. In addition, α_D is less than one, which implies diminishing returns of information supply with respect to production. In addition, the estimated θ_S and θ_I imply counter-cyclical search intensity and uncertainty.

[21] Even if data exhibit decreasing returns to scale with respect to output, the estimated posterior mean of α_D exhibits increasing returns to scale with respect to TFP, which is consistent with Farboodi and Veldkamp (2019).

Table 2: Prior and Posterior Distributions for Structural Parameters

Symbol	Description	Prior			Posterior		
		Prior Density	Mean	Std	Mode	10%	90%
		Frictions					
Ψ_K	Investment Adjustment Cost	Normal	30.00	5.00	24.938	24.862	25.010
Ψ_I	Information Matching Scale	Normal	5.00	2.00	8.006	7.994	8.014
Φ	Calvo Price Stickiness	Beta	0.5	0.15	0.324	0.287	0.365
θ_I	Information Processing Cost	Normal	0.003	0.001	2.36×10^{-6}	1.92×10^{-6}	3.02×10^{-6}
θ_S	Information Search Cost	Normal	0.003	0.001	2.02×10^{-4}	1.72×10^{-4}	2.24×10^{-4}
α_D	Returns to Data	Beta	0.50	0.20	0.704	0.677	0.725
α_S	Returns to Search Intensity	Beta	0.50	0.20	0.153	0.147	0.159
		Shock Processes					
ρ_z	TFP Shock AR	Beta	0.50	0.20	0.864	0.845	0.883
σ_z	TFP Shock std dev	Inverse-Gamma	0.10	2.00	2.359	2.088	2.626
ρ_r	Interest Rate Shock AR	Beta	0.50	0.20	0.797	0.789	0.806
σ_r	Interest Rate Shock std dev	Inverse-Gamma	0.10	2.00	0.146	0.124	0.171
ρ_I	Mutual Information Shock AR	Beta	0.50	0.20	0.957	0.943	0.977
σ_I	Mutual Information Shock std dev	Inverse-Gamma	0.10	2.00	2.327	2.084	2.497
ρ_K	Investment Shock AR	Beta	0.50	0.20	0.546	0.540	0.552
σ_K	Investment Shock std dev	Inverse-Gamma	0.10	2.00	0.997	0.921	1.065
ρ_C	Preference Shock AR	Beta	0.50	0.20	0.902	0.898	0.907
σ_C	Preference Shock std dev	Inverse-Gamma	0.10	2.00	1.949	1.678	2.151
ρ_p	Price Mark-up Shock AR	Beta	0.50	0.20	0.663	0.657	0.668
σ_p	Price Mark-up Shock std dev	Inverse-Gamma	0.10	2.00	4.405	4.191	4.640

Notes: The table shows prior and posterior distribution of the parameters that are estimated using Bayesian methods. The parameters in the top panel govern endogenous dynamics, while the parameters in the bottom panel are exogenous shock processes. The values for standard deviations (shock processes) are multiplied by 100.

5 Information Quality Driven Business Cycles

I now study the quantitative implications of information quality for business cycles. First, I quantify the effects of a fall in information quality on output and uncertainty. Second, I demonstrate that information search frictions are quantitatively important in generating time-varying information quality. Lastly, I discuss several extensions. First, I discuss the possibility of systematic mistakes when agents do not internalize time-varying information quality. Second, I consider the implications of time-varying information quality on time-varying information processing costs and their effect on price dispersion. Third, I examine policy interventions to address the fact that firms do not internalize that their production generates more data, reducing uncertainty.

5.1 Quantitative Effects of Information Quality

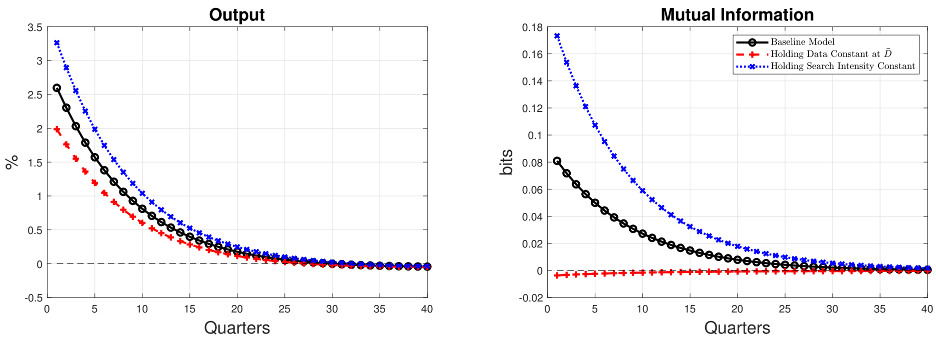
I denote the main model with rational inattention and information search frictions as the baseline model or the model with search frictions. I study the quantitative effects of information quality by comparing between $I(D_t, S_t)$ and $I(\bar{D}, S_t)$. The differences in mutual information generate different uncertainty and output dynamics. First, I examine the effects of information quality through impulse responses to a one standard deviation shock to TFP.

To decompose the effects of data (or supply of information) on output and mutual information, I set $\bar{D}$ such that $I(\bar{D}, S_{ss})$ equals the steady state of mutual information in the rational inattention model without information search frictions. I then recover the impulse responses of output and mutual information to an identical one standard deviation shock to TFP when data is fixed at $\bar{D}$. The difference between this set of impulse responses and the responses in the baseline model is attributed to fluctuations information quality and data.

In a separate exercise, I also consider impulse responses of output and mutual information when information search intensity is set to its steady state value and data is allowed to vary. The difference between this set of impulse responses and the responses in the baseline model is attributed to fluctuations in information search intensity, or information demand.

Figure 9 shows output and mutual information responses to a positive one standard deviation TFP shock. In the model with search frictions, this shock leads to a rise in output of 2.7 percent, while mutual information (measured in bits) increases by approximately 0.08 bits.

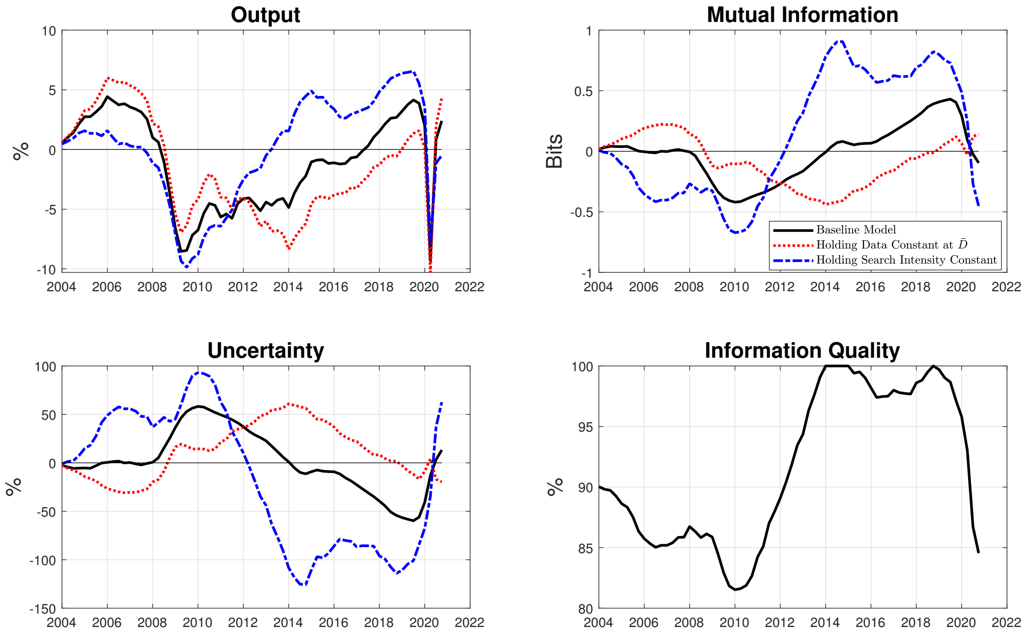
Figure 9: Quantitative Effects of Information Quality (Impulse Response Analysis)



Notes: This figure presents impulse responses of output (left panel) and mutual information (right panel) to a one standard deviation TFP shock. The black line denotes impulse responses from the model with search frictions. The green line (supply only) denotes impulse responses from the model in which information search intensity S_t is restricted to its steady state value. The red line (demand only) denote impulse responses from the model in which $D_t = \bar{D}$.

When I decompose these effects into the supply and demand of information, fluctuations in information supply generate amplified dynamics of mutual information due to pro-cyclical data. Pro-cyclical data implies pro-cyclical mutual information and counter-cyclical uncertainty, which amplifies the rise in output during booms. At the same time, fluctuations in information demand generate a decline in mutual information during booms due to counter-cyclical search intensity. Counter-cyclical search intensity dampens business cycles, leading to counter-cyclical mutual information and pro-cyclical uncertainty. As such, output rises more when considering fluctuations in the supply of information only, while output rises less when considering counter-cyclical search intensity only. Since information quality is attributed to changes in information supply, information quality accounts for approximately 20 % (0.6 percentage points) of cyclical fluctuations in output.

Figure 10: Quantitative Effects of Information Quality



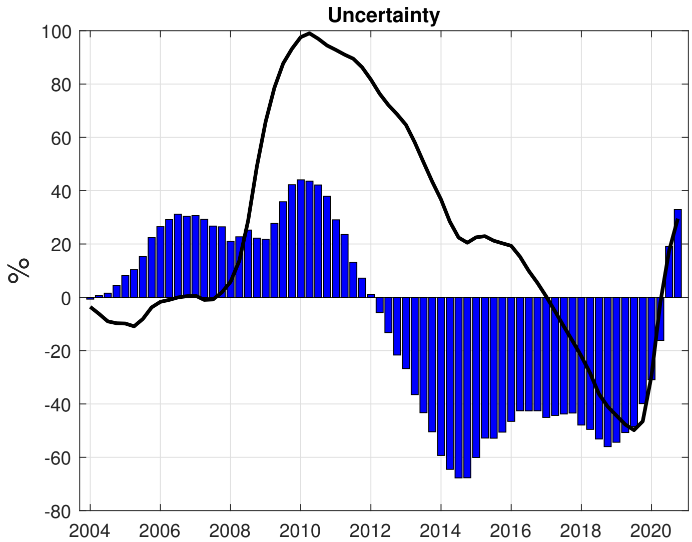
Notes: This figure presents the dynamics of output (top left panel), mutual information (top right panel), uncertainty (bottom left panel), and the model-based measure of information quality (bottom right panel) from 2004Q1 to 2021Q4. The blue line denotes estimated data from the model in which information search intensity S_t is restricted to its steady state value. The red line denote estimated data from the model in which $D_t = \bar{D}$.

Next, I look at business cycle dynamics from 2004Q1 to 2021Q4. I first recover the set of estimated shocks that generate business cycle dynamics in the baseline model. I then feed the model the same set of shocks and consider the same decomposition exercise into demand and supply of information in the impulse response analysis. As before, the difference between the baseline model and the model holding data fixed as $\bar{D}$ reveals the effect of information quality and information supply. In contrast, the difference between the baseline model and the model holding information search intensity at its steady state reveals the contribution of fluctuations in information search intensity or information demand.

In Figure 10, shifts in information demand are generated by fluctuations in information search intensity, while changes in data abundance generate fluctuations in the supply of information. When a recession occurs, data becomes scarce. At the same time, firms search for more information. Overall, the conflicting effects of these two forces lead to lower mutual information and elevated uncertainty in downturns. This is shown in Figure 10, in which mutual information declined sharply during the 2008 Financial Crisis and the 2020 recession. By definition, uncertainty rises in downturns and co-moves negatively with mutual information. Pro-cyclical mutual information and counter-cyclical uncertainty amplifies the decline in output in a crisis, as shown in the top left-hand panel of Figure 10.

By itself, counter-cyclical information search intensity (information demand) leads to relatively lower uncertainty and higher mutual information in crisis periods (red dashed line). This tends to dampen recessions. However, pro-cyclical data (information supply), leads to higher uncertainty and lower mutual information (blue dot-dashed line) in recessions. Hence, output dynamics are dampened by fluctuations in the demand for information and amplified by fluctuations in information supply.

Figure 11: Decomposing Fluctuations in Uncertainty into Information Quality



Notes: This figure presents the dynamics of uncertainty from 2004Q1 to 2021Q4. The model is estimated from uncertainty data by Jurado et al. (2015). The blue bars denote uncertainty fluctuations that originate from fluctuations in information quality.

Next, I examine the measure of information quality χ_t^{model}, which is the wedge between $I(D_t, S_t)$ (mutual information adjusted for quality) and $I(\bar{D}, S_t)$ (mutual information unadjusted for quality). In a downturn, information search intensity rises, which increases $I(\bar{D}, S_t)$. However, the decrease in data (D_t) leads to a fall in $I(D_t, S_t)$. As such, χ_t^{model} declines in a downturn. This is evident in the bottom right panel of Figure 10, which presents a decrease in this measure in 2008 and 2020. Information quality declined by about five percentage points during the 2008 Financial Crisis, constituting a 1.5 percentage point decline in output.

Figure 11 further decomposes fluctuations in uncertainty into fluctuations in the model-based measure information quality. In this exercise, I recover the differences in uncertainty between the baseline model and the model holding data fixed at $\bar{D}$. Even after accounting for uncertainty shocks, information quality fluctuations account for a substantial proportion of uncertainty fluctuations.

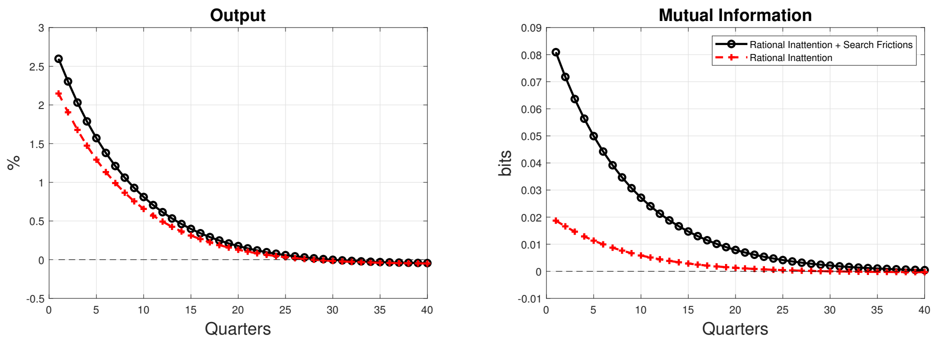
5.2 Effects of Information Search Frictions

Next, I examine the effects of information search frictions and show that information search frictions are crucial in generating time-varying information quality. The presence of information search frictions leads to poorer information quality, lower output, and higher uncertainty than a rational inattention model with pro-cyclical information acquisition. Hence, I demonstrate that introducing information search frictions is necessary to generate severe recessions, although one can obtain similar dynamics of output and uncertainty with pro-cyclical information acquisition in the rational inattention model.

I consider a model which features rational inattention only by setting θ_S equal to zero and rending equation (21) obsolete. I then compare impulse responses to the same TFP shock between both models. The differences in the impulse responses between these two models are attributed to the addition of information search frictions.

Figure 12 shows that mutual information rises about three times more in the model with search frictions compared to the rational inattention model. As such, output increases by less in the baseline model. This implies that information search frictions are quantitatively important in generating strong uncertainty multiplier dynamics.

Figure 12: Quantitative Effects of Information Quality (Impulse Response Analysis)



Notes: This figure presents the impulse responses of output (left panel) and mutual information (right panel) to a one standard deviation TFP shock. The black line represents impulse responses from the model with both rational inattention and search frictions. The red line represents impulse responses in the rational inattention model in which θ_S is set to 0.

I then feed in the same set of estimated shocks in the baseline model and recover the behavior of several macroeconomic aggregates. Figure 13 plots estimated dynamics over time and compares the baseline and rational inattention models. Mutual information is higher on average in the rational inattention model compared to the model with search frictions. This implies that uncertainty is lower on average in the rational inattention model compared to the model with search frictions. This is evident from Proposition 6, in which $\hat{I}$ (mutual information in the model with search frictions) is strictly less than I^* (mutual information in the rational inattention model). Taken together, this means that the introduction of search frictions leads to lower output on average.

Moreover, there are substantial fluctuations in mutual information and uncertainty in the model with search frictions, while the rational inattention model generates insignificant fluctuations. Output fluctuates less as a result. This is consistent with the impulse responses in Figure 12. When we consider information quality attributed to search frictions (defined as the ratio of mutual information between these two models), information quality falls by ten percentage points during both the 2008 Financial Crisis and the 2020 recession. Comparing this result with the previous section, this implies that information search frictions constitute substantially to fluctuations in information quality during recessions.

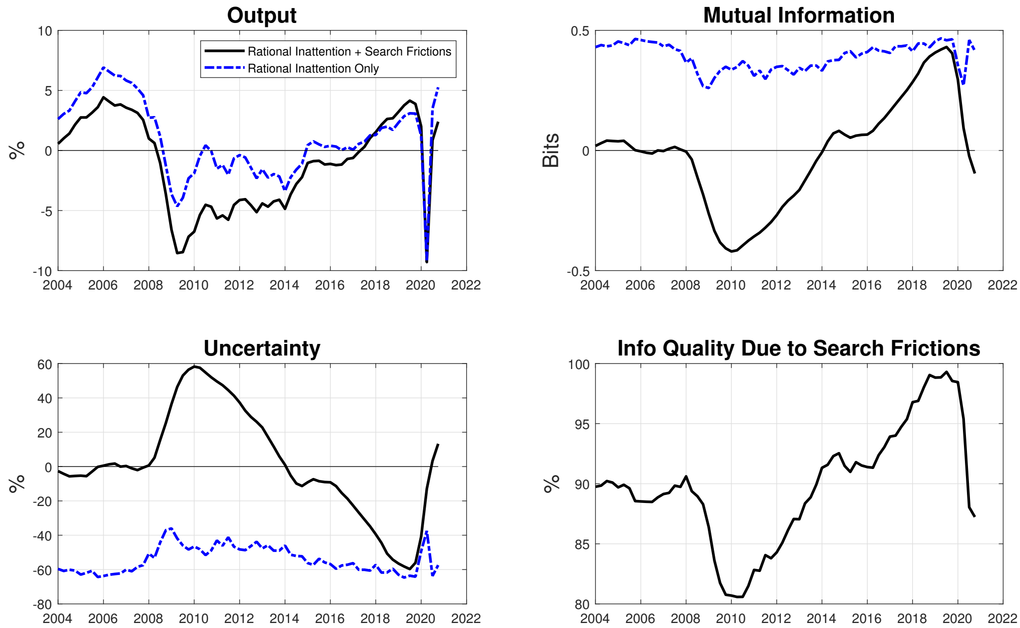
Next, I decompose uncertainty dynamics into fluctuations in information quality attributed to search frictions. I recover the model-based uncertainty measure generated by the set of estimated shocks in the model with rational inattention only and compare the differences with the baseline model. Figure 14 shows that most of the fluctuations in information quality documented in the previous exercise can be attributed to information search frictions. This implies that information search costs are quantitatively important in generating imperfect and time-varying information quality. Hence, policies that reduce information search costs can improve information quality and reduce uncertainty.

5.3 Other Extensions and Implications

Lastly, I consider some model extensions and discuss their implications.

Non-internalization of Information Quality. In the model with information search frictions, an assumption is that agents internalize that information quality is time-varying. In this scenario, they choose prices while internalizing the signal structure as:

Figure 13: Quantitative Effects of Information Search Frictions



Notes: This figure presents dynamics of output (top left panel), mutual information (top right panel), uncertainty (bottom left panel), and the model-based measure of information quality (bottom right panel) over time. The black line represents estimated data from the model with search frictions. The blue line represents estimated data in the rational inattention model which θ_S is set to 0. The dynamics of these variables are relative to the steady state values of the model with information search frictions.

$$s_{i,t}^z = \chi_t^{\text{emp}} z_t + v_{i,t} \tag{41}$$

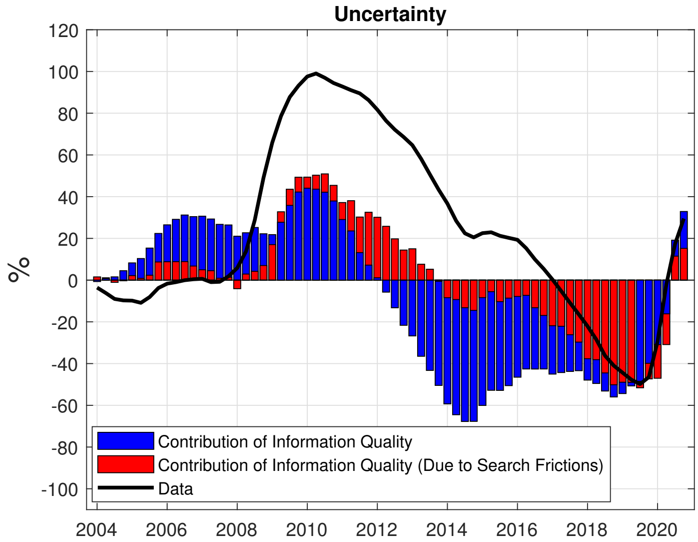
where χ_t^{emp} is a time-varying measure of information quality. Suppose that agents do not internalize fluctuations in information quality. In this scenario, they internalize the signal structure as

$$s_{i,t}^z = \chi_{ss}^{\text{emp}} z_t + v_{i,t} \tag{42}$$

where χ_{ss}^{emp} is a measure of information quality at the steady state. Even though agents internalize the signal structure given by Eq. (42), the actual signal structure is given by Eq. (41). As such, agents make systematic errors, which I deem as "mistakes". A relevant exercise is to evaluate the quantitative implications of exhibiting systematic errors.

To compute the effects of the non-internalization of information quality, I recover the decision rule of mutual information and search intensity when data is set to $\bar{D}$. I then feed the decision rule into the baseline model, in which data is allowed to

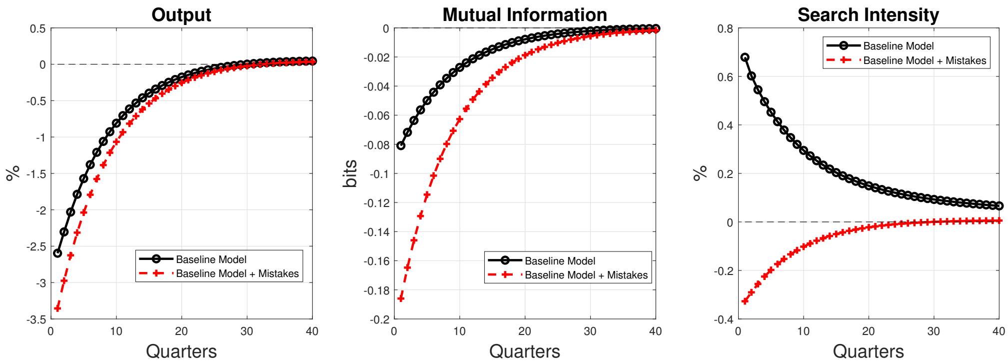
Figure 14: Decomposing Fluctuations in Uncertainty into Information Quality Due to Search Frictions



Notes: This figure presents dynamics of changes in uncertainty from 2004Q1 to 2021Q4. The model is estimated from uncertainty data by Jurado et al. (2015). The total height of the blue bars denote uncertainty fluctuations that originate from fluctuations in information quality. The total height of the red bars denote uncertainty fluctuations that originate from fluctuations in information quality due to search frictions (i.e. approximately 50 percentage points of uncertainty fluctuations are attributed to search frictions).

fluctuate. Output and mutual information dynamics generated by this decision rule are then compared with the dynamics from the baseline model.

Figure 15: Quantitative Effects of Non-internalization of Time-varying Information Quality (Impulse Response Analysis)



Notes: This figure presents impulse responses of output (left panel), mutual information (middle panel), and information search intensity (right panel) to a one standard deviation negative TFP shock.

Figure 15 compares impulse responses between the model with information search frictions and a model in which individuals do not internalize time-varying information quality. A model with non-internalization of time-varying information quality generates pro-cyclical information search intensity, as economic agents do not internalize that the information supply decreases in a downturn. As such, their perceived marginal benefit of information search intensity does not respond to fluctuations in data abundance. In fact, the marginal benefit of information search intensity decreases due to lower expected profits. Hence, these agents react to the adverse shock by reducing information search intensity. At the same time, the decrease in the supply of information amplifies the force of pro-cyclical information search intensity. As such, mutual information unambiguously declines more than the baseline model, which leads to larger losses in output.

I also document that the decline in output due to the non-internalization of information quality is related to the elasticity of substitution. As the elasticity of substitution increases, firms hold lesser monopoly power and control over market demand. When a firm makes a mistake in pricing, consumers tend to substitute away to other firms more aggressively due to the higher elasticity of substitution. As such, systematic errors and mistakes are more costly when firms hold lesser market power.

Time-Varying Information Processing Costs and Pricing. In pricing models with costly information, studies such as Woodford (2009) and Stevens (2019) formulate a rational inattention problem with information processing costs to model firms' pricing decisions. These models abstract from information search frictions introduced in this paper. Recall that the first-order condition for information acquisition in the model with search frictions satisfies:

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} = \theta_I + \theta_S Y_t^{-\alpha_D} S_t^{1-\alpha_S} \tag{43}$$

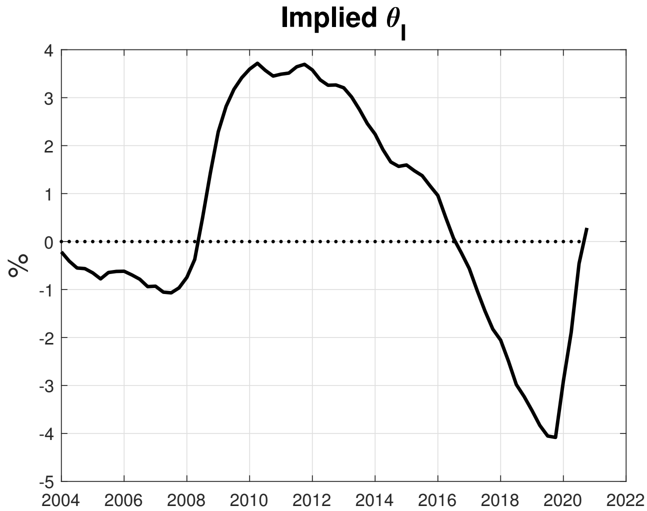
while the first order condition for information acquisition in the rational inattention model satisfies:

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} = \theta_I \tag{44}$$

Time-varying information processing costs are required for the rational inattention model to generate the same allocations as the model with search frictions. In particular, time-varying information quality implies that the model with information search frictions predicts counter-cyclical information processing costs: high (low) values of θ_I in a downturn (boom). Counter-cyclical θ_I is generated by pro-cyclical data and counter-cyclical information search intensity.

I use the estimated output and information search intensity series to back out the implied time-varying information processing costs. Figure 16 shows the implied value of θ_I rises in the 2008 and 2020 recessions and falls outside of a downturn. This is consistent with Vavra (2014), who documents these facts under counter-cyclical uncertainty. Counter-cyclical information processing costs imply that price changes occur with higher frequency and dispersion in downturns.

Figure 16: Implied Information Processing Costs



Notes: This figure presents the implied value of time-varying information processing costs in order to generate dynamics in the model with search frictions.

Policy Implications. Lastly, firms do not internalize that their production of output Y_t generates more data, improving mutual information and reducing uncertainty. This causes an externality in production.

Policies that provide subsidies on output during a recession can address this externality, and firms will produce more than usual. Hence, such a policy can correct inefficiencies that originate from the externality in production. The optimal policy will then be in the form of time-varying subsidies on output, in which the level of subsidy is larger in downturns. Intuitively, a higher level of subsidy in downturns encourages more production, which generates more data and information supply due to more sample points. Higher information supply will then reduce uncertainty, which further improves production.

6 Conclusion

This paper introduces information quality to address puzzles in real business cycle models. I find empirical evidence that even though information acquisition rises in a downturn, forecast errors rise, suggesting lower information quality in a recession.

I then build a model to explain the empirical evidence. I augment a rational inattention model with a novel ingredient: information search frictions. Information depends on the abundance of data and information search intensity. Unlike rational inattention models, which are demand-driven, I allow information supply to fluctuate and depend on the aggregate state. Time-varying information quality then depends on information supply.

The model with rational inattention and information search frictions generates counter-cyclical information acquisition and pro-cyclical information quality, which together imply counter-cyclical uncertainty. This co-movement can rationalize facts that appear at odds with each other: on the one hand, information acquisition is counter-cyclical, while on the other hand, measures of uncertainty are high, and forecasts are inaccurate in recessions.

Quantitatively, the model with information search frictions amplifies business cycle dynamics due to the cyclicality of uncertainty dynamics. Fluctuations in information quality account for a significant portion of the decline in output and rise in uncertainty. I also find that information search frictions are quantitatively important in generating time-varying information quality. In addition, information quality can also explain phenomena caused by behavioral biases, such as systematic mistakes, which can generate severe downturns.

In other extensions, the model also generates time-varying information processing costs, which explains time-varying price dispersion dynamics. Lastly, firms do not internalize that producing more output generates more information supply, which reduces uncertainty. Policies that can correct production externalities will aid in dampening recessions.

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Appendix
Information Quality Driven Business Cycles

A Data Appendix and Additional Empirical Evidence

A.1 Sticky Information Model in CG (2015) with Information Quality

This section presents an alternative interpretation in Coibion and Gorodnichenko (2015) with information quality. In the sticky information model by Mankiw and Reis (2002), individuals update their information sets each period with probability $1-\lambda$ and acquire zero information with probability λ.[22] Hence, in this case, λ is a measure of information rigidity. Mankiw and Reis (2002) show that the current period average forecast can be written as a weighted average of the previous period's average forecast and the current period's rational expectation of the variable x at time $t+h$

$$F_t x_{t+h} = (1-\lambda)E_t x_{t+h} + \lambda F_{t-1} x_{t+h} \tag{A.45}$$

where $F_t x_{t+h}$ refers to the forecast at time t of the variable x_{t+h}, and $E_t x_{t+h}$ refers to the rational expectation of the variable x at time $t+h$. Full-information rational expectations imply that

$$E_t x_{t+h} = x_{t+h} - v_{t+h,t} \tag{A.46}$$

where $v_{t+h,t}$ is the full-information rational expectations error that is orthorgonal with information at time period t or earlier. Combining Eq. (A.45) and (A.46), Coibion and Gorodnichenko (2015) obtains

$$x_{t+h} - F_t x_{t+h} = \frac{\lambda}{(1-\lambda)}(F_t x_{t+h} - F_{t-1} x_{t+h}) + v_{t+h,t} \tag{A.47}$$

In Coibion and Gorodnichenko (2015), they also run regression (A.47) for each quarter. Hence, they extract $\beta_t = \frac{\lambda_t}{(1-\lambda_t)}$ where t refers to each quarter. In this paper, I include a wedge χ_t in equation (A.45).

$$F_t x_{t+h} = (1-\lambda_t)\chi_t E_t x_{t+h} + \lambda F_{t-1} x_{t+h} \tag{A.48}$$

Recall that in the sticky information model, individuals update their information sets with probability $1-\lambda_t$. By including a wedge χ_t, this implies that even though individuals update their information sets, they do not acquire fully accurate

[22] Applications of sticky information include Auclert et al. (2020) and Carroll et al. (2020).

information, when $\chi_t \neq 1$. In this case, equation (A.48) collapses to

$$x_{t+h} - F_t x_{t+h} = \frac{1-(1-\lambda_t)\chi_t}{(1-\lambda_t)\chi_t} F_t x_{t+h} - \frac{\lambda_t}{(1-\lambda_t)\chi_t} F_{t-1} x_{t+h} + v_{t+h,t} \tag{A.49}$$

Denote $\frac{1-(1-\lambda_t)\theta_t}{(1-\lambda_t)\chi_t} = \beta_{2,t}$ and $\frac{1-(1-\lambda_t)\theta_t}{(1-\lambda_t)\chi_t} = \beta_{3,t}$. In Coibion and Gorodnichenko (2015), they restrict $\beta_{2,t} = \beta_{3,t}$. However, in this paper, I allow $\beta_{2,t}$ to differ from $\beta_{3,t}$. The two parameters $\beta_{2,t}$ and $\beta_{3,t}$ estimated from equation (A.49) allows me to identify λ_t and χ_t seperately. In particular, when information quality is high, which implies highly accurate information, then $\chi_t = 1$. This implies that $\beta_{2,t} = \beta_{3,t}$. When information quality is low, this leads to highly inaccurate information. In this case, $\chi_t \neq 1$ and $\beta_{2,t} \neq \beta_{3,t}$.

A.2 Multiplicative Wedge versus Additive Wedge

This section provides a discussion on why a multiplicative wedge in the noisy information model is preferred to an additive wedge. I find that as macroeconomic variables deviate away from their trend or steady state, forecast errors increase. To provide evidence on this phenomena, I run the following regression for GDP

$$|x_{t+h} - F_t x_{t+h}| = \beta_1 |x_t - \bar{x}| \tag{A.50}$$

The left hand side of equation (A.50) measures the absolute forecast errors, and the right hand side measures the absolute deviation of x_t away from its sample mean. I find that β_1 is positive and statistically significant at the 1 % level. This suggests that the wedge that absorbs the average expectational error across different macroeconomic variables and horizon should be multiplicative. This implies that for a given value of χ_t^{emp}, the absolute size of forecast errors rise as x_t deviates away from its steady state.

Nevertheless, I consider an additive wedge as an empirical measure of information quality. I assume that the noisy signal in Coibion and Gorodnichenko (2015) takes the following form

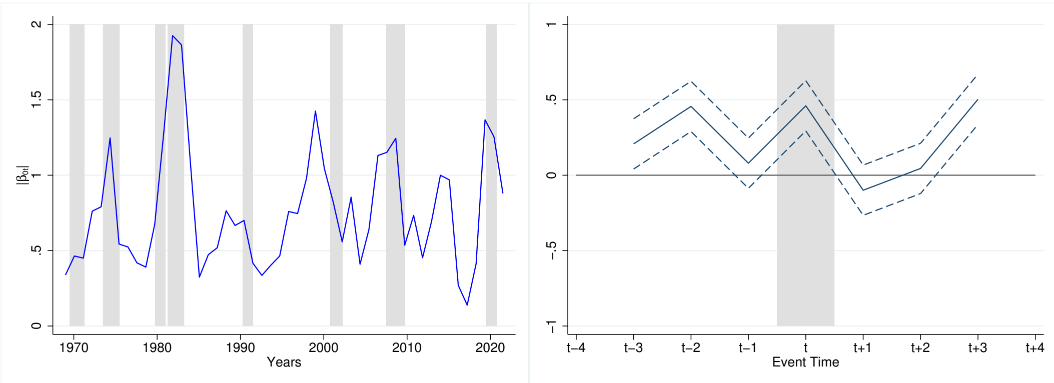
$$s_{i,t}^{z,B} = z_t + \chi_t^{\text{emp}} + v_{i,t} \tag{A.51}$$

In this scenario, χ_t^{emp} absorbs the average expectational error in Eq. (3). One can show that this collapses to the following reduced form:

$$z_{j,t+h} - F_t z_{j,t+h} = \beta_0 + \beta_1(F_t z_{j,t+h} - F_{t-1} z_{j,t+h}) + v_{j,t+h,t} \tag{A.52}$$

where β_0 is a direct measure of χ_t^{emp}. As χ_t^{emp} deviates away from zero, then β_0 deviates away from zero as well. Hence, $|\beta_0|$ is a relevant measure of information quality .The left panel of Figure 17 plots the measure over time. A recession is associated with a rise in $|\beta_0|$ and hence, a decline in information quality. The right panel of Figure 17 shows an event study analysis of $|\beta_0|$, in which period t denotes a recession, and $t-i$ and $t+i$ denote i periods before a recession and after a recession respectively. The figure implies that $|\beta_0|$ increases in a recession, which implies lower information quality in downturns.

Figure 17: Additive Wedge Measure of Information Quality



(a) $|\beta_{0,t}|$ Plot Over Time (b) Event Study of $|\beta_{0,t}$

Notes: This figure presents the measure of information quality $|\beta_{0,t}|$. Panel (a) plots the measure from 1969Q4 to 2020Q2. The measure of information rigidity is smoothed using a local average which uses Epanechnikov kernal with bandwidth equal to 0.5. Shaded in grey are the NBER recession dates. Panel (b) shows the measure of information quality using an event study approach. Dotted lines denote 95 % standard error bands.

A.3 Information Quality as Drivers of Output and Uncertainty

In this section, I provide empirical support for the following theoretical predictions. First, I show that information quality is an important contribution to uncertainty and output fluctuations. To see this, I run the following regression

$$\log Y_t = \beta_1 \log UNC_t + \beta_2 INFOQ_t \tag{A.53}$$

where $\log Y_t$ is the annualized GDP quarterly growth rate, $\log UNC_t$ is a measure of uncertainty, and $INFOQ_t$ is the measure of information quality, derived from the framework of Coibion and Gorodnichenko (2015). I use VIX and the macroeconomic uncertainty measure from Jurado et al. (2015) as measures of macroeconomic uncertainty.

Table 3: Regression Results between Output, Uncertainty and Information Quality

Dependent Variable:	$\log Y_t$	$\log Y_t$	$\log Y_t$	$\log Y_t$	$\log VIX_t$	$\log UNC_t$
	(1)	(2)	(3)	(4)	(5)	(6)
$\log VIX_t$	-7.57**	-5.51**				
	(3.59)	(2.46)				
$\log UNC_t$			-21.08	-7.87		
			(14.92)	(12.24)		
$INFOQ_t$		-4.66***		-4.44***	0.033*	0.053***
		(1.32)		(1.63)	(0.019)	(0.008)
Observations	211	211	211	211	211	211
Adjusted R^2	0.043	0.257	0.087	0.209	0.0148	0.157

Notes: Robust standard errors are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively.

Table 3 shows the result. In panel (1), the coefficient on VIX is significant at the 5 % level, while the coefficient on the uncertainty measure from Jurado et al. (2015) is statistically insignificant. Nevertheless, both measures of uncertainty are negatively correlated with GDP growth rates.

Upon introducing the measure of information quality, the coefficients on both uncertainty measures fall in absolute magnitude. Moreover, the coefficient on $INFOQ_t$ is statistically significant at the 1% level in panels (2) and (4). In addition, the regressions in panel (2) and (4) suggests that lower information quality (higher value of $INFOQ_t$) is associated with lower GDP growth. This suggests that some of the effects of uncertainty on output originates from information quality and thus, information quality accounts for some fluctuations in output.

Panels (5) and (6) shows that lower information quality is associated with higher uncertainty. This is consistent with the theoretical framework in which time-varying information quality accounts for fluctuations in uncertainty.

A.4 Information Quality and the Uncertainty Multiplier

Next, I examine the effects of information quality on the uncertainty multipler. I run the following regression.

$$\log Y_t = \beta_1 \log UNC_t + \beta_2 INFOQ_t + \beta_3 INFOQ_t \times \log UNC_t \tag{A.54}$$

The uncertainty multiplier generates amplification if information quality is lower. Hence, I expect β_3 to be negative, as the correlation between uncertainty and output

Table 4: Regression Results between Output, Uncertainty and Information Quality

Dependent Variable:	$\log Y_t$ (1)	$\log Y_t$ (2)
$\log VIX_t$	2.45	
	(2.67)	
$\log UNC_t$		2.94
		(13.04)
$INFOQ_t$	20.83***	-5.13***
	(3.03)	(0.93)
$INFOQ_t \times \log UNC_t$		-23.56***
		(3.01)
$INFOQ_t \times \log VIX_t$	-18.24***	
	(2.07)	
Observations	211	211
Adjusted R^2	0.343	0.372

Notes: Robust standard errors are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively.

should be stronger when information quality falls (a rise in the variable $INFOQ_t$).

Table 4 shows the results. β_3 is negative and statistically significant across two different uncertainty measures. This is consistent with the theoretical prediction that the amplification feedback loop between output and uncertainty is stronger when information quality is lower.

B Detailed Derivations of the Model

B.1 Derivation of the Production Function

The production function of firm i can be written as:

$$Y_{i,t} = Z_t N_{i,t} \tag{B.55}$$

Recall that the demand function is:

$$Y_{i,t} = P_{i,t}^{-\epsilon} Y_t \tag{B.56}$$

and the optimal price setting solution is

$$P_{i,t} = \frac{\epsilon}{\epsilon - 1} E_{i,t}\big(\frac{1}{Z_t}\Big| s_{i,t}\big) W_t \tag{B.57}$$

Combine (B.55), (B.56), and (B.57) to obtain

$$\Big(\frac{\epsilon}{\epsilon - 1}\Big)^{-\epsilon} \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big| s_{i,t}\Big)\Big]^{-\epsilon} [W_t]^{-\epsilon} Y_t = (\frac{K_t}{N_t})^{\alpha} N_{i,t} \tag{B.58}$$

The next step is to find an expression for W_t in terms of Z_t,

Zero profit condition in the final goods sector implies

$$\int_0^1 P_{i,t} Y_{i,t} di = Y_t \tag{B.59}$$

Substituting (B.55) into (B.59) yields

$$\int_0^1 P_{i,t}^{1-\epsilon} di = 1 \tag{B.60}$$

Substituting (B.57) into (B.60) yields

$$W_t = \Big(\frac{\epsilon - 1}{\epsilon}\Big)\Big(\int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big| s_{i,t}\Big)\Big]^{1-\epsilon} di\Big)^{\frac{1}{\epsilon-1}} \tag{B.61}$$

Substitute (B.61) into (B.58) yields

$$\Big(\int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big| s_{i,t}\Big)\Big]^{1-\epsilon} di\Big)^{\frac{\epsilon}{\epsilon-1}} \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big| s_{i,t}\Big)\Big]^{-\epsilon} \Big(\frac{1}{Z_t}\Big) Y_t = (\frac{K_t}{N_t})^{\alpha} N_{i,t} \tag{B.62}$$

Now integrate both sides of (B.62)

$$\Big(\int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{1-\epsilon}di\Big)^{\frac{\epsilon}{\epsilon-1}}\int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big(\frac{1}{Z_t}\Big)Y_t di = \int_0^1 N_{i,t}di \tag{B.63}$$

Observe that

$$\int_0^1 \Big[E_i\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big(\frac{1}{Z_t}\Big)di = E_{i,t}\Big[\Big(\frac{1}{Z_t}\Big)\Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big]di \tag{B.64}$$

By Law of Iterated Expectations,

$$\int_0^1 \Big[E_i\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big(\frac{1}{Z_t}\Big)di = E_{i,t}\Bigg\{E_{i,t}\Bigg[\Big(\frac{1}{Z_t}\Big)\Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big|s_{i,t}\Bigg]\Bigg\}di \tag{B.65}$$

$$\int_0^1 \Big[E_i\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big(\frac{1}{Z_t}\Big)di = E_{i,t}\Bigg\{\Big[E_{i,t}\Big(\frac{1}{Z_t}\Big)\Big|s_{i,t}\Big]\Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Bigg\}di \tag{B.66}$$

$$\int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{-\epsilon}\Big(\frac{1}{Z_t}\Big)di = \int_0^1 \Big[E_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big]^{1-\epsilon}di \tag{B.67}$$

Use (B.63) into (B.62) and $\int_0^1 N_{i,t}di = N_t$, I obtain

$$Y_t = \Bigg(\int_0^1 \mathbb{E}_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)^{1-\epsilon}\mathrm{di}\Bigg)^{\frac{1}{\epsilon-1}}N_t \tag{B.68}$$

.

$$\begin{aligned}\mathbb{E}_{i,t}\Big(\frac{1}{Z_{i,t}}\Big|s_{i,t}\Big)^{1-\epsilon} &= exp\Big(-(1-\epsilon)\mathbb{E}_{i,t}(Z_{i,t}|s_{i,t}) + \frac{1}{2}(1-\epsilon)Var(Z_{i,t}|s_{i,t})\Big) &\text{(B.69)}\\ &= exp\Big(-(1-\epsilon)\hat{z} + \frac{1}{2}(1-\epsilon)\frac{1}{\tau_{v,i,t}+\tau_z})\Big) &\text{(B.70)}\end{aligned}$$

$$\int_0^1 \mathbb{E}_{i,t}\Big(\frac{1}{Z_t}\Big|s^z_{i,t}\Big)^{1-\epsilon} di = \mathbb{E}_i\Big[exp\Big(- (1-\epsilon)\hat{z}_i + \frac{1}{2}(1-\epsilon)\frac{1}{\tau_{v,i,t}+\tau_z})\Big)\Big] \tag{B.71}$$

$$= exp\Big(- (1-\epsilon)\mathbb{E}_i(\hat{z}_i) + \frac{1}{2}(\epsilon-1)^2 Var_i(\hat{z}_i) + \frac{1}{2}(1-\epsilon)\frac{1}{\tau_{v,t}+\tau_z})\Big) \tag{B.72}$$

$$= exp\Big((\epsilon-1)\hat{z} + \frac{1}{2}(\epsilon-1)^2\frac{\tau_{v,t}}{\tau_z}\frac{1}{\tau_{v,t}+\tau_z} + \frac{1}{2}(1-\epsilon)\frac{1}{\tau_{i,t}+\tau_z}) \tag{B.73}$$

$$\Big(\int_0^1 \mathbb{E}_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)^{1-\epsilon}\text{di}\Big)^{\frac{1}{\epsilon-1}} = exp\Big(\hat{z} + \frac{1}{2}(\epsilon-1)\frac{\tau_{v,i,t}}{\tau_z}\frac{1}{\tau_{v,t}+\tau_z} - \frac{1}{2}\frac{1}{\tau_{v,t}+\tau_z}\Big) \tag{B.74}$$

$$= \exp\Big(\hat{z} - \frac{1}{2\tau_z} + \frac{1}{2\tau_z}\epsilon\frac{1}{1+\frac{\tau_z}{\tau_{v,t}}}\Big) \tag{B.75}$$

Constant Learning Gain Approximation:

In this subsection, I show that the average posterior expectation with time-varying Kalman gain can be approximated into an average posterior expectation with a constant learning gain.

$$\mathbb{E}_i\Big[\mathbb{E}_{i,t}\Big(\frac{1}{Z_t}\Big|s_{i,t}\Big)\Big] = \frac{\tau_{v,t}}{\tau_{v,t}+\tau_z}z_t = (1-2^{-2I_t})z_t \tag{B.76}$$

$$\approx (1-2^{-2I_{ss}})(z_t - z_{ss}) + 2\log 2 I_{ss} z_{ss}(I_t - I_{ss}) \tag{B.77}$$

$$= (1-2^{-2I_{ss}})(z_t) \tag{B.78}$$

$$= \bar{g}z_t = \hat{z} \tag{B.79}$$

$$\tag{B.80}$$

since z_{ss} equals 0. Hence, the average posterior expectations across all agents i with time varying Kalman gain can be approximated with an average posterior expectations with constant learning gain where $\bar{g} = 1 - 2^{-2I_{ss}}$.

B.2 Log-Linearization

B.2.1 Rational Inattention Model

Rational Inattention Model (Deriving Steady State).

Consider the case in which $N_t = 1$.[23] The equation below is the production equation.

$$Y_t = \exp\Big(\hat{z} - \frac{1}{2\tau_z} + \frac{1}{2\tau_z}\epsilon\frac{1}{1+\frac{\tau_z}{\tau_{v,t}}}\Big) \tag{B.81}$$

where $\hat{z}$ is the posterior expectations. Expressing this in terms of mutual information, this is equivalent to

$$Y_t = \exp\Big(\hat{z} - (1-\epsilon)\frac{1}{2\tau_z} - \epsilon\frac{2^{-2I_t}}{2\tau_z}\Big) \quad \text{(Production)} \tag{B.82}$$

Firm's realized profits after stage 3 is given by:

$$\Pi_{i,t} = \Bigg(P_{i,t} - \frac{1}{Z_t}C(W_t, R_t)\Bigg)Y_{i,t} = \frac{1}{\epsilon}Y_t\frac{\mathbb{E}_{i,t}(\frac{1}{Z_t})^{1-\epsilon}}{\int_0^1 \mathbb{E}_{i,t}(\frac{1}{Z_t})^{1-\epsilon}\mathrm{di}} \tag{B.83}$$

After taking expectations of Eq. (B.83) over all possible signal realizations, firm i's expected profit is given by:

$$\Pi^E_{i,t} = \frac{1}{\epsilon}Y_t\frac{\exp\Big\{(\epsilon-1)(\hat{z}-(1-\epsilon)\frac{1}{2\tau_z} - \frac{2^{-2I_t}}{2\tau_z})\Big\}}{\exp\Big\{(\epsilon-1)(\hat{z}-(1-\epsilon)\frac{1}{2\tau_z} - \frac{2^{-2I_{-i,t}}}{2\tau_z})\Big\}} \tag{B.84}$$

Differentiating Eq. (B.84) with respect to I_t yield

$$\frac{\partial\Pi^E_{i,t}}{\partial\theta_I} = \frac{\epsilon-1}{\epsilon}\frac{Y_t\cdot 2\log 2}{2^{2I_t}\cdot 2\tau_z}\frac{\exp\Big\{(\epsilon-1)(\hat{z}-(1-\epsilon)\frac{1}{2\tau_z} - \frac{2^{-2I_t}}{2\tau_z})\Big\}}{\exp\Big\{(\epsilon-1)(\hat{z}-(1-\epsilon)\frac{1}{2\tau_z} - \frac{2^{-2I_{-i,t}}}{2\tau_z})\Big\}} \tag{B.85}$$

Imposing symmetry and equating $I_t = I_{-i,t}$ yields

$$\frac{\partial\Pi^E_{i,t}}{\partial\theta_I} = \frac{\epsilon-1}{\epsilon}\frac{Y_t\cdot 2\log 2}{2^{2I_t}\cdot 2\tau_z} \tag{B.86}$$

The information acquisition equation in the rational inattention model becomes

[23] I hold N_t constant due to the assumption of inelastic labor supply in Section 3.

$$\frac{\partial \Pi^E_{i,t}}{\partial \theta_I} = \frac{\epsilon - 1}{\epsilon}\frac{2\log 2 \cdot Y_t}{2\tau_z \cdot 2^{2I_t}} = \theta_I \quad \text{(Information Acquisition)} \tag{B.87}$$

The following equations charaterize the steady state (an explicit analytical solution cannot be derived due to the functional form that contains I_{ss}):

$$Y_{ss} = \exp\Big(- (1-\epsilon)\frac{1}{2\tau_z} - \frac{2^{-2I_{ss}}}{2\tau_z}\Big) \quad \text{(Production)} \tag{B.88}$$

$$\frac{\epsilon - 1}{\epsilon}\frac{2\log 2 \cdot Y_{ss}}{2\tau_z \cdot 2^{2I_{ss}}} = \theta_I \quad \text{(Information Acquisition)} \tag{B.89}$$

The Rational Inattention Model (Log-Linearization).

Production Equation:

Let me start with

$$Y_t = \exp\Big(\hat{z} - \frac{2^{-2I_t}}{2\tau_z}\Big) \quad \text{(Production)} \tag{B.90}$$

Denote $y_t = \log Y_t$. Taking logs on both sides yield:

$$y_t = \hat{z} - \frac{2^{-2I_t}}{2\tau_z} \tag{B.91}$$

Taking a Taylor's expansion around y_{ss} and I_{ss} in (B.91) yields

$$y_t - y_{ss} = \bar{g}(z_t - z_{ss}) + \frac{2\log 2 \cdot 2^{-2I_{ss}}}{2\tau_z}(I_t - I_{ss}) \tag{B.92}$$

where $\bar{g}$ is the constant learning Kalman gain and $\hat{z} = \bar{g}z_t$. This yields

$$\tilde{y}_t = \eta_1 \tilde{I}_t + \eta_2 \tilde{z}_t \quad \text{(Production)} \tag{B.93}$$

where $\tilde{y}_t = y_t - y_{ss}$, $\tilde{z}_t = z_t - z_{ss}$, $\tilde{I}_t = I_t - I_{ss}$, $\eta_1 = \frac{2\log 2 \cdot 2^{-2I_{ss}}}{2\tau_z}$, and $\eta_2 = \bar{g}$.

Information Acquisition Equation:

Now consider

$$\frac{\epsilon - 1}{\epsilon}\frac{2\log 2 \cdot Y_t}{2\tau_z \cdot 2^{2I_t}} = \theta_I \quad \text{(Information Acquisition)} \tag{B.94}$$

Denote $A_1 = \frac{\epsilon-1}{\epsilon}\frac{2\log 2}{2\tau_z}$. Taking logs on both sides yield:

$$\log A_1 + y_t - 2\log 2 \cdot I_t = \log \theta_I \tag{B.95}$$

Linearizing (B.95) around y_{ss} and I_{ss} yields

$$\tilde{y}_t - 2\log 2 \cdot \tilde{I}_t = 0 \tag{B.96}$$

which is equivalent to

$$\tilde{y}_t - \phi_1 \tilde{I}_t = 0 \quad \text{(Information Acquisition)} \tag{B.97}$$

Where $\phi_1 = 2\log 2$.

B.2.2 Rational Inattention + Information Search Frictions

The following equations characterizes I_t, Y_t and S_t.

$$Y_t = \exp\Big(\hat{z} - \frac{2^{-2I_t}}{2\tau_z}\Big) \quad \text{(Production)} \tag{B.98}$$

$$\frac{\epsilon - 1}{\epsilon}\frac{2\log 2 \cdot Y_t}{2\tau_z \cdot 2^{2I_t}} = \theta_I + \theta_s \exp(-\alpha_D z_t) S_t^{1-\alpha_S} \quad \text{(Information Acquisition)} \tag{B.99}$$

$$I_t = \exp(\alpha_D z_t) S_t^{\alpha_S} \quad \text{(Information Matching Function)} \tag{B.100}$$

Steady State.

Y_{ss}, I_{ss} and S_{ss} jointly satisfies (an explicit analytical solution cannot be derived due to the functional form that contains I_{ss})

$$Y_{ss} = \exp\Big(- \frac{2^{-2I_{ss}}}{2\tau_z}\Big) \quad \text{(Production)} \tag{B.101}$$

$$\frac{\epsilon - 1}{\epsilon}\frac{2\log 2 \cdot Y_{ss}}{2\tau_z \cdot 2^{2I_{ss}}} = \theta_I + \theta_s S_{ss}^{1-\alpha_S} \quad \text{(Information Acquisition)} \tag{B.102}$$

$$I_{ss} = S_{ss}^{\alpha_S} \quad \text{(Information Matching Function)} \tag{B.103}$$

Making Use of a General Result of Log-Linearization.

Before log-linearizing the model with search frictions, I make use of the following observation that

$$\log(C_1 + C_{2,t}) - \log(C_1 + C_{2,ss}) \approx \frac{C_{2,ss}}{C_1 + C_{2,ss}}[\log(C_{2,t}) - \log(C_{2,ss})] \tag{B.104}$$

The purpose of this result in (B.111) is to log-linearize the right hand side of (B.99).

Proof. Consider the following equation.

$$C_t = C_1 + C_{2,t} \tag{B.105}$$

Then

$$\log(C_t) = \log(C_1 + C_{2,t}) \tag{B.106}$$

I linearize equation (B.106) around C_{ss} and $C_{2,ss}$

$$\log(C_t) - \log(C_{ss}) = \log(C_1 + C_{2,t}) - \log(C_1 + C_{2,ss}) \approx \frac{1}{C_1 + C_{2,ss}}(C_{2,t} - C_{2,ss}) \tag{B.107}$$

Notice that

$$\log(C_{2,t}) - \log(C_{2,ss}) \approx \frac{(C_{2,t} - C_{2,ss})}{C_{2,ss}} \tag{B.108}$$

This yields

$$[\log(C_{2,t}) - \log(C_{2,ss})] \cdot C_{2,ss} \approx (C_{2,t} - C_{2,ss}) \tag{B.109}$$

Substituting (B.109) into (B.107) tields

$$\log(C_1 + C_{2,t}) - \log(C_1 + C_{2,ss}) \approx \frac{C_{2,ss}}{C_1 + C_{2,ss}}[\log(C_{2,t}) - \log(C_{2,ss})] \tag{B.110}$$

which is equivalent to

$$\widetilde{C_1 + C}_{2,t} = \frac{C_{2,ss}}{C_1 + C_{2,ss}} \tilde{C}_{2,t} \tag{B.111}$$

$\square$

Rational Inattention + Information Search Frictions (Log-Linearization).

Production Equation:

Log-linearizing the production equation (B.98) yields (B.93), identical to the rational inattention model.

Information Acquisition Equation:

I now log-linearize (B.99). Taking logs on both sides of (B.99) yields

$$\log A_1 + y_t - 2\log 2 \cdot I_t = \log[\theta_I + \theta_s \text{exp}(-\alpha_D z_t) S_t^{1-\alpha_S}] \tag{B.112}$$

Log-linearizing (B.112) yields

$$\tilde{y}_t - 2\log 2 \cdot \tilde{I}_t = \log[\theta_I + \theta_s \text{exp}(-\alpha_D z_t) S_t^{1-\alpha_S}] - \log[\theta_I + \theta_s \text{exp}(-\alpha_D z_{ss}) S_{ss}^{1-\alpha_S}] \quad \text{(B.113)}$$

Making use of the result in (B.111) yields

$$\tilde{y}_t - 2\log 2 \cdot \tilde{I}_t = \frac{\theta_s S_{ss}^{1-\alpha_S}}{\theta_I + \theta_s S_{ss}^{1-\alpha_S}} \Big\{ \log[\theta_s \text{exp}(-\alpha_D z_t) S_t^{1-\alpha_S}] - \log[\theta_s \text{exp}(-\alpha_D z_{ss}) S_{ss}^{1-\alpha_S}] \Big\} \quad \text{(B.114)}$$

This reduces to

$$\tilde{y}_t - 2\log 2 \cdot \tilde{I}_t = \frac{\theta_s S_{ss}^{1-\alpha_S}}{\theta_I + \theta_s S_{ss}^{1-\alpha_S}} \Big\{ -\alpha_D \tilde{z}_t + (1-\alpha_S)\tilde{s}_t \Big\} \quad \text{(B.115)}$$

where $\tilde{s}_t = \log S_t - \log S_{ss}$. Equation (B.115) then becomes

$$\tilde{y}_t - \phi_1 \tilde{I}_t = -\phi_2 \tilde{z}_t + \phi_3 \tilde{s}_t \quad \text{(Information Acquisition)} \quad \text{(B.116)}$$

Where $\phi_2 = \frac{\theta_s S_{ss}^{1-\alpha_S}}{\theta_I + \theta_s S_{ss}^{1-\alpha_S}} \alpha_D$ and $\phi_3 = \frac{\theta_s S_{ss}^{1-\alpha_S}}{\theta_I + \theta_s S_{ss}^{1-\alpha_S}} (1-\alpha_S)$

Information Matching Equation:

Next, I log-linearize the information matching function. Taking logs on both sides of (B.100) yield

$$\log I_t = \alpha_D z_t + \alpha_S s_t \quad \text{(B.117)}$$

Linearizing (B.117) yields

$$\log I_t - \log I_{ss} = \alpha_D \tilde{z}_t + \alpha_S \tilde{s}_t \quad \text{(B.118)}$$

This is approximately equal to

$$\frac{I_t - I_{ss}}{I_{ss}} = \alpha_D \tilde{z}_t + \alpha_S \tilde{s}_t \quad \text{(B.119)}$$

which reduces to

$$\tilde{I}_t = \psi_1 \tilde{z}_t + \psi_2 \tilde{s}_t \quad \text{(Information Matching Function)} \quad \text{(B.120)}$$

where $\tilde{I}_t = I_t - I_{ss}$, $\psi_1 = I_{ss}\alpha_D$ and $\psi_2 = I_{ss}\alpha_S$

B.3 Proof of Propositions

Proposition 1. *Equilibrium mutual information and output, as functions of* z_t*, are given by:*

$$\tilde{I}_t = \frac{1}{\phi_1 - \eta_1}\eta_2\tilde{z}_t \tag{B.121}$$

$$\tilde{y}_t = \frac{\phi_1}{\phi_1 - \eta_1}\eta_2\tilde{z}_t \tag{B.122}$$

where ϕ_1*,* η_1*, and* $\eta_2 > 0$*.*

Proof. The log-linearized equations are

$$\tilde{y}_t - \phi_1\tilde{I}_t = 0 \quad \text{(Information Acquisition)} \tag{B.123}$$

$$\tilde{y}_t = \eta_1\tilde{I}_t + \eta_2\tilde{z}_t \quad \text{(Production)} \tag{B.124}$$

Combining (B.123) and (B.124) yields (B.121) and (B.122).

□

Corollary 1. *Define the uncertainty multiplier as* $\frac{\partial \tilde{y}_t}{\partial \eta_2 \tilde{z}_t} = \frac{\phi_1}{\phi_1 - \eta_1}$*. If mutual information is pro-cyclical* ($\phi_1 > 0$)*, then the uncertainty multiplier is greater than 1. If mutual information is counter-cyclical* ($\phi_1 < 0$)*, then the uncertainty multiplier is less than 1.*

Proof. Corollary 1 comes immediately from (B.122).

□

Proposition 2. *Equilibrium mutual information and search intensity, as functions of* z_t*, are given by:*

$$\tilde{s}_t = B_t^s\tilde{z}_t = \frac{\eta_1\psi_1 + \eta_2 - \phi_1\psi_1 + \phi_2}{\phi_3 + \phi_1\psi_2 - \eta_1\psi_2}\tilde{z}_t \tag{B.125}$$

$$\tilde{I}_t = B_t^I\tilde{z}_t = \frac{\eta_2\psi_2 + \phi_2\psi_2 + \psi_1}{\phi_1\psi_2 + \phi_3 - \phi_1\psi_2\eta_1}\tilde{z}_t \tag{B.126}$$

where ϕ_1*,* ϕ_2*,* ϕ_3*,* η_1*,* η_2*,* ψ_1*, and* $\psi_2 > 0$*.*

Proof. The log-linearized equations are

$$\tilde{y}_t - \phi_1\tilde{I}_t = -\phi_2\tilde{z}_t + \phi_3\tilde{s}_t \quad \text{(Information Acquisition)} \tag{B.127}$$

$$\tilde{y}_t = \eta_1 \tilde{I}_t + \eta_2 \tilde{z}_t \quad \text{(Production)} \tag{B.128}$$

$$\tilde{I}_t = \psi_1 \tilde{z}_t + \psi_2 \tilde{s}_t \quad \text{(Information Matching Function)} \tag{B.129}$$

Substituting (B.128) into (B.127), I obtain

$$\eta_1 \tilde{I}_t + \eta_2 \tilde{z}_t - \phi_1 \tilde{I}_t = -\phi_2 \tilde{z}_t + \phi_3 \tilde{s}_t \tag{B.130}$$

Substituting (B.129) into (B.130), I obtain

$$(\eta_1 - \phi_1)(\psi_1 \tilde{z}_t + \psi_2 \tilde{s}_t) + \eta_2 \tilde{z}_t = -\phi_2 \tilde{z}_t + \phi_3 \tilde{s}_t \tag{B.131}$$

Rearranging (B.131) yields (B.125). Substituting (B.125) into (B.129) yields (B.126).

□

Corollary 2. *Mutual information is pro-cyclical, that is,* $\frac{\partial \tilde{I}_t}{\partial \tilde{z}_t} < 0$*. Suppose that* $\phi_1 > \frac{\eta_1 \phi_2 + \eta_2 + \phi_2}{\psi_1}$*, then information search intensity is counter-cyclical, that is,* $\frac{\partial \tilde{s}_t}{\partial \tilde{z}_t} < 0$*.*

Proof. Pro-cyclical mutual information comes from (B.126).

$$\frac{\partial \tilde{s}_t}{\partial \tilde{z}_t} = B_t^s = \frac{\eta_1 \psi_1 + \eta_2 - \phi_1 \psi_1 + \phi_2}{\phi_3 + \phi_1 \psi_2 - \eta_1 \psi_2} \tag{B.132}$$

$$B_t^s < 0 \iff \phi_1 > \frac{\eta_1 \phi_2 + \eta_2 + \phi_2}{\psi_1} \tag{B.133}$$

□

Proposition 3. *Suppose that $I(\bar{D}, S_t)$ satisfies*

$$s^z_{i,t} = z_t + v_{i,t} \tag{B.134}$$

Then there exist a random variable χ^{emp}_t, such that $\hat{I}_t$ satisfies

$$s^z_{i,t} = \underbrace{\chi^{emp}_t z_t}_{\text{Realization Component}} + \underbrace{v_{i,t}}_{\text{Noise Component}} \tag{B.135}$$

where χ^{emp}_t has finite variance $\sigma^{\chi,emp}_t$ and mean equal to one.

Proof. Consider χ^{emp}_t to be a random variable with the following properties:

$$\chi^{emp}_t = \begin{cases} 1 & \text{with probability } p \\ \sqrt{1+K} & \text{with probability } \frac{1-p}{2} \\ \sqrt{1-K} & \text{with probability } \frac{1-p}{2} \end{cases} \tag{B.136}$$

where $K > 0$.[24]

Then $\sigma^{\chi,emp}_t$ is increasing in K.

Denote the signal in (B.134) as s^z_1 and signal in (B.135) as s^z_2. First, I show that

$$Var(z_t|s^z_2) > Var(z_t|s^z_1) \tag{B.137}$$

By the Law of Total Variance,

$$Var(z_t|s^z_2) = E\big(Var(z_t|s^z_2, \chi^{emp}_t)\big) + Var\big(E(z_t|s^z_2, \chi^{emp}_t)\big) \tag{B.138}$$

From (B.138), I compute $Var(z_t|s^z_2, \chi^{emp}_t))$

$$Var(z_t|s^z_2, \chi^{emp}_t) = p\frac{\sigma^2_v\sigma^2_z}{\sigma^2_v + \sigma^2_z} + \frac{1-p}{2}\frac{\sigma^2_v\sigma^2_z}{(1-K)\sigma^2_v + \sigma^2_z} + \frac{1-p}{2}\frac{\sigma^2_v\sigma^2_z}{(1+K)\sigma^2_v + \sigma^2_z} \tag{B.139}$$

$$= p\frac{\sigma^2_v\sigma^2_z}{\sigma^2_v + \sigma^2_z} + (1-p)\sigma^2_v\sigma^2_z\frac{1}{\sigma^2_z + \sigma^2_v - K^2\frac{\sigma^2_z}{\sigma^2_z+\sigma^2_v}} \tag{B.140}$$

Observe from (B.140) that $Var(z_t|s^z_2, \chi^{emp}_t)$ is monotonically increasing in K and $\sigma^{\chi,emp}_t$, and $Var\big(E(z_t|s^z_2, \chi^{emp}_t)\big)$ is independent of K and $\sigma^{\chi,emp}_t$. This implies that from (B.138), $Var(z_t|s^z_2)$ is monotonically increasing in K and $\sigma^{\chi,emp}_t$. Since I^* satisfies

[24] The purpose of the square root is for analytical tractability. Note that $\sqrt{1+K} \approx 1 + \frac{1}{2}K$ and $\sqrt{1-K} \approx 1 - \frac{1}{2}K$.

$Var(z_t|s_1^z)$ and $\hat{I} < I^*$, then there exists K and $\sigma_t^{\chi,emp}$ such that $\hat{I}_t$ satisfies

$$s_{i,t}^z = \chi_t^{\text{emp}} z_t + v_{i,t} \tag{B.141}$$

□

Proposition 4. *Assume that χ_t^{emp} is a random variable with $\sigma_t^{\chi,emp}$ and mean equal to one. Then $\sigma_t^{\chi,emp}$ is decreasing in χ_t^{model}.*

Proof.

$$\hat{I}_t = \chi_t^{\text{model}} I(\bar{D}, S_t) \tag{B.142}$$

Differentiating Eq. (B.142) yields

$$\frac{\partial \hat{I}_t}{\partial \sigma_t^{\chi,emp}} = \frac{\partial \chi_t^{\text{model}}}{\partial \sigma_t^{\chi,emp}} I_t^* \tag{B.143}$$

$$\frac{\partial \hat{I}_t}{\partial \sigma_t^{\chi,emp}} = \frac{\partial \hat{I}_t}{\partial Var(z_t|s_2^z)} \frac{\partial Var(z_t|s_2^z)}{\partial \sigma_t^{\chi,emp}} > 0 \tag{B.144}$$

Hence,

$$\frac{\partial \chi_t^{\text{model}}}{\partial \sigma_t^{\chi,emp}} = \frac{\partial \hat{I}_t}{\partial \sigma_t^{\chi,emp}} \frac{1}{I_t^*} > 0 \tag{B.145}$$

□

Proposition 5. *If $\frac{\partial \tilde{S}_t}{\partial \tilde{z}_t} < 0$, then χ_t^{model} is strictly increasing in z_t.*

Proof.

$$\hat{I}_t = \chi_t^{\text{model}} I(\bar{D}, S_t) \tag{B.146}$$

Differentiating (B.146) yields

$$\frac{\partial \hat{I}_t}{\partial z} = \frac{\partial \chi_t^{\text{model}}}{\partial z} I(\bar{D}, S_t) + \frac{\partial I(\bar{D}, S_t)}{\partial z} \chi_t^{\text{model}} \tag{B.147}$$

Differentiating (B.146) yields

$$\frac{\partial \hat{I}_t}{\partial z} - \frac{\partial I(\bar{D}, S_t)}{\partial z} \chi_t^{\text{model}} = \frac{\partial \chi_t^{\text{model}}}{\partial z} I(\bar{D}, S_t) \tag{B.148}$$

Since $\frac{\partial \hat{I}_t}{\partial z} > 0$ and $\frac{\partial I(\bar{D}, S_t)}{\partial z} < 0$, it follows that $\frac{\partial \chi_t^{\text{model}}}{\partial z} > 0$. □

Proposition 6. *When $\theta_S > 0$, Mutual information in the model with RI and search frictions is strictly lesser that its counterpart in the rational inattention model, that is,*

$$\hat{I}_t < I_t^* \tag{B.149}$$

where $\hat{I}_t$ and I_t^* are mutual information obtained in the model with rational inattention and search frictions and the rational inattention respectively.

Proof. Mutual Information in the rational inattention model satisfies

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} = \theta_I \tag{B.150}$$

Mutual Information in the model with search frictions satisfies

$$\frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} = \theta_I + \theta_S z_t^{-\alpha_s} I_t^{\alpha_S} \tag{B.151}$$

Define

$$f^*(I_t) = \frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} - \theta_I \tag{B.152}$$

$$\hat{f}(I_t) = \frac{\partial \Pi_t^E}{\partial I(z_t; s_t^z)} - \theta_I - \theta_S z_t^{-\alpha_s} I_t^{\alpha_S} \tag{B.153}$$

Optimality implies

$$f^*(I^*) = \hat{f}(\hat{I}) = 0 \tag{B.154}$$

(B.154) implies that

$$f^*(I^*) - f^*(\hat{I}) = \hat{f}(\hat{I}) - f^*(\hat{I}) < 0 \tag{B.155}$$

Since $f^*(I_t)$ is decreasing in I_t, it follows that $\hat{I} < I^*$.

□

C Details on the Estimated Version of the Model

C.1 Model

I extend the model in Section 3 to a quantitative version that I estimate on US data. I add various frictions and shocks to bring the framework close to a New Keynesian Model.

C.1.1 Firms

The problem of the final good producer is identical to Section 3, with the exepction that ϵ_t is now stochastic and subject to price markup shocks. Each intermediate good firms now face the following production function:

$$Y_{i,t} = Z_t K_{i,t}^{\alpha} N_{i,t}^{1-\alpha} \tag{C.156}$$

Each firm's cost minimization implies that

$$W_t N(Y_{i,t}) + R_t K(Y_{i,t}) = \frac{1}{Z_t}(\frac{W_t}{1-\alpha})^{1-\alpha}(\frac{R_t}{\alpha})^{\alpha} \tag{C.157}$$

In addition, the intermediate good firms now face Calvo pricing frictions. Denote $C(W_t, R_t) = (\frac{W_t}{1-\alpha})^{1-\alpha}(\frac{R_t}{\alpha})^{\alpha}$. Each firm maximize:

$$\max_{P_{i,t}} \mathbb{E}_{i,t}\left[\sum_{s=0}^{\infty} \zeta^s \left(P_{i,t} - \frac{C(W_t, R_t)}{Z_t}\right) P_{i,t}^{-\epsilon} Y_t | \mathcal{I}_{i,t}\right] \tag{C.158}$$

The optimality condition satisfies

$$\frac{P_{i,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_{i,t}\left[\sum_{s=0}^{\infty} \zeta^s \left(\frac{C(W_{t+s}, R_{t+s})}{Z_{t+s}}\right) P_{i,t}^{\epsilon} Y_t | \mathcal{I}_{i,t}\right]}{\mathbb{E}_{i,t}\left[\sum_{s=0}^{\infty} \zeta^s P_{i,t}^{\epsilon-1} Y_t | \mathcal{I}_{i,t}\right]} \tag{C.159}$$

where $P_t = [\zeta P_{t-1}^{1-\epsilon} + (1-\zeta) P_{i,t}^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$.

C.1.2 Households

Representative Household. The household consumes, supplies labor, accumulate bonds, and hold shares in firms. They maximize:

$$\max_{C_t,N_t} \sum_{t=0}^{\infty} \beta^t [\log C_t - \phi \frac{N_t^{1+\eta}}{1+\eta}]$$

Their budget constraint is

$$C_t + \frac{B_{t+1}}{(1+r_t)P_t} + K_{t+1} + \frac{\Psi_t^K}{2}\Big(\frac{K_{t+1}-K_t}{K_t}\Big)^2 K_{t+1} = W_t N_t + R_t K_t + (1-\delta)K_t + \Pi_t \quad \text{(C.160)}$$

where C_t is consumption, B_t is the stock of nominal bonds a household enters the period with, r_t is the nominal interest rate earned on bonds, K_t is the stock of capital a household enters the period with, W_t is the wage rate, R_t is the rental rate of capital, and Π_t is the profit remitted by firms. Households also face capital adjustment costs $(\Psi_t^K > 0)$. The first order conditions are:

$$U_{c,t} = E_t[U_{c,t+1}\beta(1+r_t)\frac{P_t}{P_{t+1}}] \quad \text{(C.161)}$$

$$\frac{W_t}{C_t} = \phi N_t^{\eta} \quad \text{(C.162)}$$

$$U_{c,t}\Big(1+\frac{\Phi_t^K}{2}(\frac{K_{t+1}-K_t}{K_t})^2+\Phi_t^K\frac{K_{t+1}}{K_t}(\frac{K_{t+1}-K_t}{K_t})\Big) = E_t[U_{c,t+1}\beta\Big(R_{t+1}+1-\delta+\Phi_{t+1}^K(\frac{K_{t+2}-K_{t+1}}{K_{t+1}})\Big)] \quad \text{(C.163)}$$

which equates the real returns of bonds and capital with the stochastic discount factor, and $U_{c,s}$ denote the marginal utility of consumption in period s.

C.1.3 Monetary Policy

There is a Taylor rule specified as

$$\frac{1+r_t}{1+\bar{r}} = \Big[\frac{1+r_{t-1}}{1+\bar{r}}\Big]^{\rho_R}\Big[(\frac{\pi_t}{\bar{\pi}})^{\nu_1}(\frac{Y_t}{\bar{Y}})^{\nu_2}\Big]v_t \quad \text{(C.164)}$$

such that interest rates react to deviations of inflation from steady state and deviations of output from steady state. $\rho_r > 0$ captures interest rates smoothing and

v_t is a stochastic disturbance that captures monetary shocks.

C.1.4 Structural Shocks

1. **Total Factor Productivity Shock**

$$\log Z_t = (1-\rho_z)\log \bar{Z} + \rho_z \log Z_{t-1} + e_{Z,t} \tag{C.165}$$

2. **Monetary Policy Shock**

$$\frac{1+r_t}{1+\bar{r}} = \Big[\frac{1+r_{t-1}}{1+\bar{r}}\Big]^{\rho_R}\Big[(\frac{\pi_t}{\bar{\pi}})^{\nu_1}(\frac{Y_t}{\bar{Y}})^{\nu_2}\Big]v_t \tag{C.166}$$

$$\log v_t = (1-\rho_v)\log \bar{v} + \rho_v \log v_{t-1} + e_{v,t} \tag{C.167}$$

3. **Preference Shock**

$$\log \gamma_t = (1-\rho_\gamma)\log \bar{\gamma} + \rho_\gamma \log \gamma_{t-1} + e_{\gamma,t} \tag{C.168}$$

4. **Price Mark-up Shock**

$$\log \epsilon_t = (1-\rho_\epsilon)\log \bar{\epsilon} + \rho_\epsilon \log \epsilon_{t-1} + e_{\epsilon,t} \tag{C.169}$$

5. **Investment Technology Shock**

$$\log \Psi_t^K = (1-\rho_K)\log \bar{\Psi}^K + \rho_K \log \Psi_{t-1}^K + e_{K,t} \tag{C.170}$$

6. **Mutual Information/Uncertainty Shock**

$$\log \Psi_t^I = (1-\rho_I)\log \bar{\Psi}^I + \rho_I \log \Psi_{t-1}^I + e_{I,t} \tag{C.171}$$

C.2 Observables and mapping between the data and the model

I use the following data collected from FRED and other sources for the estimation. The data period is from 2004Q1 to 2020Q2.

1. Output
   - Model: $\tilde{Y}_t^{\text{obs}} = \log\left(\frac{Y_t}{Y_{t-1}}\right)$
   - Data: Nominal GDP (FRED,GDP), divided by GDP deflator (FRED, GDPDEF) and population (FRED, B230RC0Q173SBEA), log-transformed, first-differenced and de-meaned.
2. Consumption
   - Model: $\tilde{C}_t^{\text{obs}} = \log\left(\frac{C_t}{C_{t-1}}\right)$
   - Data: Real consumption expenditures of nondurable goods and services (FRED: PCNDGC96 and PCESVC96), divided by population (FRED, B230RC0Q173SBEA), log-transformed, first-differenced and de-meaned.
3. Investment
   - Model: $I\tilde{N}V_t^{\text{obs}} = \log\left(\frac{INV_t}{INV_{t-1}}\right)$
   - Data: Sum of nominal gross private domestic investment expditures (FRED: GPDI) and nominal private consumption expenditures on durable goods (FRED: PCDG), divided by GDP deflator (FRED, GDPDEF) and population (FRED, B230RC0Q173SBEA), log-transformed, first-differenced and de-meaned.
4. Inflation Rate
   - Model: $\tilde{\pi}_t^{\text{obs}} = \log\left(\frac{\pi_t}{\pi}\right)$
   - Data: Log difference of GDP Implicit Price Deflator (FRED, GDPDEF) minus 0.5 percentage point.
5. Interest Rate
   - Model: $\tilde{i}_t^{\text{obs}} = \log\left(\frac{R_t}{R}\right)$
   - Data: Nominal effective Federal Funds Rate (FRED: FEDFUNDS), divided by 400 to express in quarterly units.

6. Information Search Intensity
    - Model: $\tilde{S}_t^{\text{obs}} = \log\left(\frac{S_t}{S_{t-1}}\right)$
    - Data: Average Google Search Shares of 20 US media outlets in the Business and Industrial category, log-transformed, first-differenced and de-meaned.
7. Uncertainty
    - Model: $U\tilde{N}C_t^{\text{obs}} = \log\left(\frac{Var(z|s^z)_t}{Var(z|s^z)_{t-1}}\right)$
    - Data: Uncertainty Index from Jurado et al. (2015), log-transformed, first-differenced and de-meaned.

C.3 Comparison of Estimated Uncertainty Series

In this section, I compare uncertainty that is estimated from various model specifications.

Information Quality Can Explain Uncertainty Shocks.

First, I show that a fraction of uncertainty shocks can be attributed to time-varying information quality. As shocks to mutual information can be mapped directly to uncertainty, I retrieve uncertainty shocks for the baseline model and the model in which data is fixed at $\bar{D}$. The difference between estimated uncertainty shocks for the two models is then attributed to information quality.

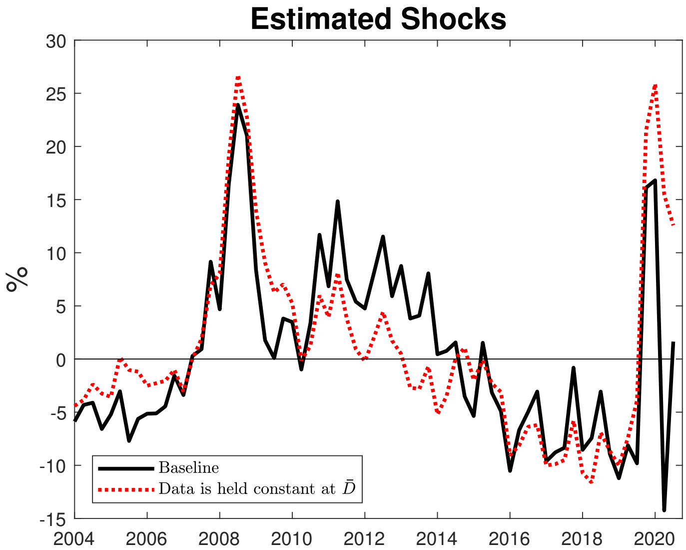
Figure 18 shows that the model, which does not account for information quality (data fixed at $\bar{D}$, red dotted line), generates large uncertainty shocks in the 2008 financial crisis and the 2020 Coronavirus recessions. In contrast, in tranquil times, estimated uncertainty shocks are relatively muted.

When I introduce information quality in the model, uncertainty shocks in recessions are smaller, implying that time-varying information quality can explain a fraction of large uncertainty shocks in downturns. During tranquil times, uncertainty shocks in the model with information quality are more significant than the model without information quality on average. This implies that better information quality explains favorable uncertainty shocks in booms.

Information Search Frictions and Estimated Uncertainty.

Second, I examine the role of information search frictions and its implications on estimated uncertainty. I retrieve the uncertainty series generated by the rational inattention model. The left panel of Figure 19 shows that the uncertainty series generated by the rational inattention model lacks large fluctuations in uncertainty. Hence, it does not match the empirical series of uncertainty well.

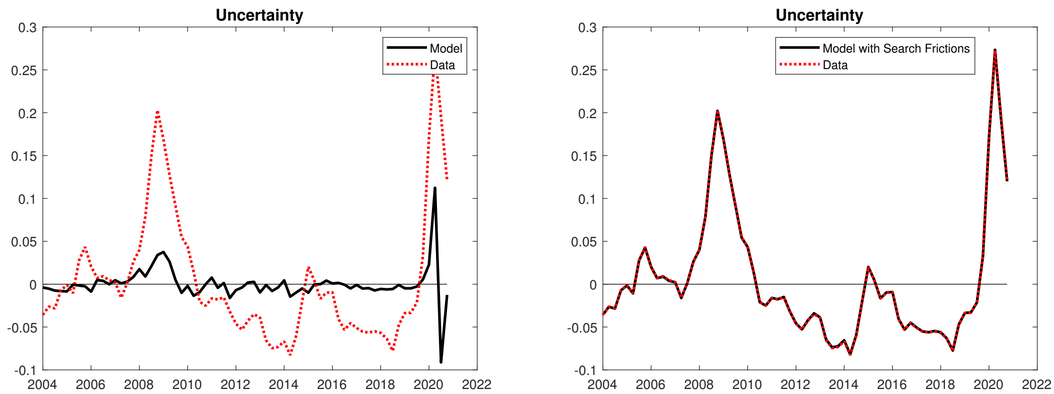
Figure 18: Uncertainty Shocks (Baseline Model vs Model (Data Fixed at $\bar{D}$))



Notes: This figure presents dynamics estimated uncertainty shocks from 2004Q1 to 2021Q4. The black line denote estimated uncertainty shocks in the baseline model. The red dotted line denote estimated uncertainty shocks in the model in which data is held constant at $\bar{D}$. The difference between the black and red dotted line is attributed to endogenous information quality.

However, when I introduce endogenous information quality and information search frictions, the estimated uncertainty series matches the empirical series of uncertainty perfectly, even though I assume that uncertainty is contaminated with measurement error in the estimation process. This implies that information search frictions play a key role in matching uncertainty dynamics.

Figure 19: Estimated $\%\Delta$ in Uncertainty



(a) RI Model (b) RI + Information Search Frictions

Notes: This figure presents the dynamics of estimated uncertainty in the rational inattention model (left hand panel) and the model with both rational inattention and information search frictions (right hand panel). The red dotted lines denote the empirical measure of uncertainty, while the black lines are the model implied uncertainty.